Institutionen för matematik
KTH
Chaotic Dynamical Systems, SF2720, Fall 2012

Homework assignment 2

The exercises are due on October 26, 2012

(1) Assume that a finite sequence of digits is given (for example, 123456789). Show (by writing down the details) that there always exists an integer $n > 0$ such that the decimal expansion of 2^n begins with this sequence (e.g., there exists n such that $2^n = 123456789....$).

(2) Do Exercise 4.3.4 on page 135

(3) Do Exercise 4.3.5 (you only need to prove the assertions of Proposition 4.3.5 and 4.3.8, not 4.3.9).

(4) Do Exercise 4.1.8 on page 108.

(5) Let $f : S^1 \to S^1$ be an orientation-reversing homeomorphism. Show that f must have two fixed points.

(6) Let $T : [-1, 1] \to [-1, 1]$ be give by $T(x) = 1 - 2|x|$. Does T exhibit sensitive dependence on initial conditions? Is T chaotic (definition 7.2.1 on page 205)? Is T topologically mixing?

(7) Let $T(x, y) = (2x, 3y)$ (mod1) be a map of the torus. Show that T is topologically mixing, that its periodic points are dense, and find the number of periodic points of (not necessarily prime) period n for each $n \geq 1$.