Homework 1: The definition

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1. Let d be a non-negative integer and put $P = \left\{(x, y) \in \mathbb{Z} \times \mathbb{Z}; x^2 - dy^2 = 1\right\}$. Let (x_0, y_0) and (x_1, y_1) be elements in P. Consider the following operation
$$(x_0, y_0) \cdot (x_1, y_1) = (x_0 x_1 + d y_0 y_1, x_0 y_1 + x_1 y_0).$$
Show that $\cdot$ is a binary operation on P and that P endowed with this operation is a group.
2. Let H be the following set of matrices
$$\left\{\left(\begin{array}{cc} a & -b \\ b & a \end{array}\right); a, b \in \mathbb{R} \text{ and } a^2 + b^2 \neq 0\right\}.$$
Show that H endowed with the usual matrix multiplication is a group.
3. Let $\mathbb{C}^*$ be the group consisting of the non-zero complex numbers together with the usual multiplication of complex numbers and let H be as in exercise 2. Show that the map
$$\varphi : \mathbb{C}^* \to H$$
given by
$$a + ib \mapsto \left(\begin{array}{cc} a & -b \\ b & a \end{array}\right)$$
is an isomorphism.