SF2729 Groups & Rings

Homework 2: Subgroups and involutions

1. If all proper subgroups of a group are abelian, is it the case that the group itself is also abelian?
2. Cauchy's theorem states that if a prime p divides the order of a finite group G, then G has an element of order p. Show Cauchy's theorem in the case of $p = 2$; i.e., show that if a finite group has even order, then it has an element of order 2. An element of order 2 is called an *involution*. (Hint: Each element has a unique inverse.)
3. Let G be a group and suppose that $a \in G$ is the unique element of order 2. Show that a commutes with all elements in G, that is, show that the equality $ax = xa$ is true for all elements $x \in G$. (Hint: What is the order of xax^{-1}?)