SF2729 GROUPS & RINGS

HOMEWORK 7: p-GROUPS AND SYLOW'S THEOREMS

1. Let G be a group and p a prime. Show that if $G/Z(G)$ is cyclic, then G is abelian. Conclude that a group of order p^2 must be abelian. (Hint: Recall that for a p-group, $Z(G) \neq 1$.)
2. Show that a group of order 56 has a proper normal subgroup.
3. Let A_5 be the alternating group of degree five. Calculate the number of Sylow-p groups of A_5 for $p = 3$ and $p = 5$.
4. **Bonus/voluntary:** Continue with exercise 1, and show that for a non-abelian group G of order p^3, where p is a prime, it must be the case that $[G, G] = Z(G) \cong \mathbb{Z}/p$.