

SF2729 GROUPS AND RINGS
HOMEWORK 8: RINGS, INTEGRAL DOMAINS, AND EULER'S
THEOREM

DUE: JANUARY 18, HAND IN WITH ORNELLA GRECO

Problem 1. Let R be a commutative ring. An element $a \in R$ is called *nilpotent* if some power of it is 0. Show that the subset $\text{Nil}(R)$ of nilpotent elements of R is a (non-unital) subring.

Problem 2. Let R and S be commutative, unital rings of characteristic m, n , respectively. What is the characteristic of the ring $R \times S$, with the ring structure defined by componentwise addition and multiplication?

Problem 3. Show that every number of the form $n^{13} - n$ for $n \in \mathbf{Z}$ is divisible by 30.

Problem 4 (bonus problem). Compute the greatest common divisor of all numbers of the form

$$n^{13} - n \quad (n \in \mathbf{Z}).$$