SF2729 GROUPS AND RINGS
HOMEWORK 9: FIELDS, POLYNOMIAL RINGS, FACTORIZATION

DUE: JANUARY 24, HAND IN WITH ORNELLA GRECO OR GIVE TO ME IN CLASS

Problem 1. Let $\phi\colon R \to K = \mathrm{Quot}(R)$ be the canonical map from a integral domain R to its field of quotients. Show that if K is finite then R was already a field, and ϕ is an isomorphism.

(Hint: Thm. 19.11 might help; you needn't reproduce that proof, but make sure you understand it.)

Problem 2. Denote by R the set of all polynomials in $f(x) \in \mathbf{Q}[x]$ with the property that $f(n) \in \mathbf{Z}$ for all $n \in \mathbf{Z}$. Show that R is a unital subring of $\mathbf{Q}[x]$ and give an example of a polynomial in R which is not in $\mathbf{Z}[x]$.

Problem 3. Use long division of polynomials to determine the greatest common divisor of $x^5 - x^2 - x - 1$ and $x^4 - x^2 + 1$ in $\mathbf{Z}/3\mathbf{Z}[x]$.