SF2729 GROUPS AND RINGS
HOMEWORK 10: IRREDUCIBLE POLYNOMIALS, IDEALS, PRIME IDEALS AND MAXIMAL IDEALS

DUE: JANUARY 31, HAND IN WITH ORNELLA GRECO

Problem 1. Write the polynomial $f(x) = 8x^4 - 2x^3 - 15x^2 + 33x - 24 \in \mathbf{Z}[x]$ as a product of irreducible factors. Explain why those factors are irreducible.

Problem 2. Given a homomorphism of commutative rings $\phi\colon R \to S$, show that

(1) if $I \triangleleft R$ is an ideal then $\phi(I) \triangleleft S$ is an ideal in $\phi(R)$ but not necessarily in all of S. (Give a counterexample for the latter statement.)
(2) if $J \triangleleft S$ is an ideal then $\phi^{-1}(J) \triangleleft R$ is an ideal.

Problem 3. The *radical* of a unital, commutative ring R is defined to be the intersection of all maximal ideals in R. Show that the radical of R consists precisely of those $x \in R$ for which $1 - xy$ is a unit for all $y \in R$.