SF2729 GROUPS AND RINGS
HOMEWORK 11: PRINCIPAL IDEAL DOMAINS AND UNIQUE FACTORIZATION DOMAINS

DUE: FEBRUARY 7, HAND IN WITH ORNELLA GRECO

Problem 1. Show that in a PID R, every proper ideal $I \triangleleft R$ is contained in a maximal ideal.

Problem 2 (bonus problem)**.** Show that the condition that R is a PID is not needed: it works for left ideals in any ring R. (You will have to use Zorn's lemma.)

You can skip Problem 1 if you do this problem.

Problem 3. Find the factorization into irreducible elements of $x^3 - y^3$ in $\mathbf{Q}[x, y]$ and prove that each factor is irreducible.

Problem 4. Show that the ring $\mathbf{Z}[\sqrt{-3}]$ consisting of all complex numbers that can be written as $a + b\sqrt{-3}$ with a, $b \in \mathbf{Z}$, is not a UFD. (Be careful about showing that elements are irreducible!)