Homework 1

Galois theory

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The solutions are to be handed in no later than Friday, 1st of March. Please pay attention to the presentation as well as the arguments given in the solutions.

Exercise 1. (1 credit). Let $\mathbf{C}$ be the field of complex numbers and $K \subset \mathbf{C}$ be the splitting field of $f(x) = x^3 - 2$ over the field of rational numbers $\mathbf{Q}$. Find a complex number z such that $K = Q(z)$.

Exercise 2. (1 credit). Let F be a field of characteristic p and $f(x) = x^p - x - c$ be a polynomial in $F[x]$. Show that f is either irreducible in $F[x]$ or that all its roots lie in F. Give an example of F and c so that f is irreducible.

Exercise 3. (1 credit). Let $F \subset K \subset E$ be field extensions. Assume that $F \neq K$ and there is $x \in E$ such that $E = F(x)$. Show that E is algebraic over K.