

## Homework 2

### Galois theory

Examinator: Wojciech Chachólski

The solutions are to be handed in no later than Friday, 22nd of March. Please pay attention to the presentation as well as the arguments given in the solutions.

**Exercise 1. (1 credit).** Let  $\mathbf{Q}$  be the field of rational numbers. Consider  $f = X^4 + X^2 + 1$  in  $\mathbf{Q}[X]$ . Determine the splitting field of  $f$ .

**Exercise 2. (1 credit).** Let  $F$  be a finite field. Show that any algebraic extension  $F \subset E$  is separable.

**Exercise 3. (1 credit).** Let  $p$  be a prime number,  $\mathbf{F}_p$  the field with  $p$ -elements, and  $\mathbf{F}_p(X)$  the field of rational functions with coefficients in  $\mathbf{F}_p$ . Construct an extension  $\mathbf{F}_p(X) \subset E$  which is not separable.