Exam instructions for SF2705 Fourier Analysis.

Reading: The exam will cover chapter 1-7 in Stein-Shakarchi *"Fourier Analysis: an Introduction"*.

Marks and grades: The exam will consist of six questions the first two worth 12 marks each and the last four questions worth 10 marks totaling 64 marks.
38 marks or more will give a final grade C.
47 marks or more will give a final grade B.
52 marks or more will give you a B with the possibillity to give an oral exam that might give you an A.

Questions that might be asked. The exam could be divided into two parts. The first two questions consists of 6 smaller questions each. The small questions are short and require short answers, usually stating a theorem or a definition. Some of the short questions only require a "yes" or "no" as an answer. Rarely they need a calculation.
Expect questions of the type:

Example question 1: *Define the Fouriertransform of a function* $f \in \mathcal{S}(\mathbb{R}^n)$. **2 marks**

Example question 2: Let $f \in \mathcal{S}(\mathbb{R}^3)$ be given. How many solutions $u(x_1, x_2, x_3, t)$, such that $u(x_1, x_2, x_3, t) \in \mathcal{S}(\mathbb{R}^3)$ for each $t > 0$, can be found to the following initial value problem

$$\begin{aligned} &\frac{\partial^2 u}{\partial t^2} = \Delta u && \text{in } \mathbb{R}^3 \times (0, \infty) \\ &u(x_1, x_2, x_3, 0) = f(x_1, x_2, x_3) && \end{aligned}$$

[You do not have to prove your answer.] **(2 marks)**

The short questions gives us a chance to cover more material in the exam.

The four final questions will be more theoretical. They are worth 10 marks each. They will ask you to prove something. It is very difficult to exactly delimit what you are allowed to assume and what you need to prove in the question formulations. This is especially true for theorems in the later chapters when the proof of a theorem might rely on several previous results. I have tried to be as clear as I can when formulating the questions.
A typical proof question might look like this:

Example question 3: *Prove that if* $f(x)$ *be continuous on the circle and differentiable at* $x = 0$ *then* $S_N(f)(0) \to f(0)$ *as* $N \to \infty$.

[You may assume the Riemann-Lebesgue Lemma and any property of $D_N(x)$ without proof.] **(10 marks)**

The remark is an attempt to clarify what you may assume. In this particular case the proof uses the Riemann-Lebesgue Lemma and that $\int_{-\pi}^{\pi} D_N(x)dx = 1$ - but that is not what you are supposed to prove.
However, no clarification like the remark after the question will clarify everything. You will need to use your judgment. For instance the following is a true theorem about the Dirichlet kernel:

True Theorem 1: *It is a property of the Dirichlet kernel* $D_N(x)$ *that if* f *is a continuous function on the circle that is differentiable at* $x = 0$ *then* $D_N * f(0) \to f(0)$.

But if you use that theorem in your solution of example question 3 I am not likely to give you many marks. Since assuming that theorem would make the question into a triviality. Naturally I will try to exclude such loop-holes from the exam. But I will probably miss some loop-holes. But if you come up with a two line proof of a 10 mark question chances are that you are assuming something that was not intended. As a rule of thumb I want proofs similar to those in the course book with the same or similar assumptions.
Many proofs will need some results from classical analysis. For instance that every Cauchy sequence of real numbers have a convergent subsequence. This is not a course in classical analysis and, therefore, you do not have to prove those results. However, when you use a result from analysis you should mention that! If you want to use, for instance, that continuous functions are Riemann integrable you do not have to prove that but you should mention something like *"Since continuous functions are Riemann integrable the following expression is well defined..."* This shows an awareness of the theory that this course is building upon.
Once again it is impossible to clearly delimit what you may assume and what you have to prove. You will have to use your judgment.

Of the four proof questions I deem the last two (that is question 5 and 6 on the exam) to be somewhat more difficult.
Questions during the exam: Besides the impossibility to be exactly define what you may assume the exam probably contains some typos, mistakes, unclarities, confusions, badly worded questions and other unwelcome things. I will therefore visit the exam three times (after around 30 minutes, 2 hours and maybe $3\frac{1}{2}$ hours) to answer any questions that you might have.

Thoughts about marks and grades: The first two exam questions are simple and I expect all of you to get most marks on those. They are there to force you to read the entire material (many short questions allow us to cover more material) and to force you to memorize the basic definitions and results. They test knowledge on the E and D grade level.

Question 3-4 involves proofs of simpler and more straightforward theorems. They are supposed to test for a C. In particular, question 1-4 gives at most 44 marks and a C level student should be able to get some marks on question 5-6 as well.

Question 5-6 are somewhat more difficult and are there to test for a B and (possibly) an A. For a B one should get most of the proofs essentially right.

Oral Exam: We will try to mark the exam as fast as we can. As soon as we have the results we will contact you to let you know if you are given the possibility to take the oral exam. I hope that we should be able to have the oral exams on Monday the 9th.

Solution to Example question 2: The solution to example question 2 is: *Infinitely many solutions.* This since we may find a unique solution to

$$\begin{aligned} &\tfrac{\partial^2 u}{\partial t^2} = \Delta u && \text{in } \mathbb{R}^3 \times (0,\infty) \\ &u(x_1,x_2,x_3,0) = f(x_1,x_2,x_3) && \\ &\tfrac{\partial u(x_1,x_2,x_3,0)}{\partial t} = g(x_1,x_2,x_3) && \end{aligned}$$

for any $g \in \mathcal{S}(\mathbb{R}^3)$. So there exist a solution for each $g \in \mathcal{S}(\mathbb{R}^3)$ and there are infinitely many such g.