Assignments Week 10 SF2705 Fourieranalysis.

These are the things that you are expected to do before the Lecture on the **15th of April**.

1 Reading: Read chapter 5.1.6 to 5.2, pages 142-153 in Stein-Shakarchi.

2 Discussion questions. Let us assume that all functions in these discussion questions are in $\mathcal{S}(\mathbb{R})$.

1. In Proposition 1.2 on page 136 we proved that the Fourier transform of $f'(x)$ was $2\pi i\xi\hat{f}(\xi)$ and Plancherel's formula (Proposition 1.11 on page 142) states that $\widehat{f*g}(\xi) = \hat{f}(\xi)\hat{g}(\xi)$. Use this to (heuristically) show that if

$$\sum_{k=0}^{n} a_k \frac{\partial^k u(x)}{\partial x^k} = f(x)$$

then

$$u(x) = \left(\widehat{\frac{1}{p(2\pi i\xi)}}\right) * f(x)$$

where $p(x) = \sum_{k=0}^{n} a_k x^k$. In particular, we may find an expression for the solution of any ordinary differential equation with constant coefficients.

By a Heuristic argument I mean an argument where you just assume that every expression is well defined etc.

2. In mathematics we are always on the prow for interesting generalizations. A particularly common way to create generalizations is to find more general ways to express the same concept. Take for instance $\widehat{f^{(k)}} = (2\pi i\xi)^k \hat{f}(\xi)$ which, using the Fourier inversion formula

$$f^{(k)}(x) = \int_{-\infty}^{\infty} (2\pi i\xi)^k \hat{f}(\xi) e^{2\pi i x\xi} d\xi.$$

Can you use this to define the r :th order derivatives of f for any $r \in \mathbb{R}$ such that $r \geq 0$?

Can one show that $\frac{\partial^{1/2}}{\partial x^{1/2}}\left(\frac{\partial^{1/2} f(x)}{\partial x^{1/2}}\right) = \frac{\partial f(x)}{\partial x}$? Or more generally that $\frac{\partial^r}{\partial x^r}\left(\frac{\partial^s f(x)}{\partial x^s}\right) = \frac{\partial^{r+s} f(x)}{\partial x^{r+s}}$ for any $r, s \geq 0$.

3. During the first lecture we remarked that it was extremely difficult to solve a partial differential equation such as $\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}$ since it involves infinitely many relations, one for each (x,t). We also remarked that in contrast it was rather easy to solve an ordinary differential equation since the derivative for a function defined on $\mathbb{R}$ has an inverse - the integral.

Keep this in mind and reread page 146 in Stein-Shakarchi. Can you based on this formulate why the Fourier transform is important for PDE theory?

3 Problems to consider: Do **6, 7** and **8** in Stein-Shakarchi pp. 161-162.

4. Assignment for the 15th of April: Hand in a solution of exercise **9** on p. 163 in Stein-Shakarchi on Tuesday the 15th of April.

5 Office hours: It does not seem to be any need for office hours. In case you have any pressing question please write me an email (johnan@kth.se) and we can book a time on Friday.

6 Exam date is set: The exam will be on Wednesday the 4th of June 14:00-19:00. You will have to register for the exam using the course web "my pages" ("mina sidor") between the 14th of April and the 4th of May. If you can not register for the exam online (most likely if you are an SU or PhD student) then you have to go to the student expedition on the first floor in this building and fill in a registration form.