Assignments Week 2 SF2705 Fourieranalysis.

These are the things that you are expected to do before the Lecture on the **28th of January**.

1 Reading: Read the following in Stein-Shakarichi

- Chapter 2.3-2.5 pp 44-58

2 Discussion questions.

1. We write $f(x) \sim \sum_{n=-\infty}^{\infty} a_n e^{i\frac{2\pi nx}{L}}$ with a " $\sim$" and not a " $=$" for the formal Fourier series. Why?
2. What is the link between Corollary 2.4 and section 2.4, are they related?
3. What does Corollary 5.4 imply for the theory of Fourier series.
4. There are three types of kernels introduced in this chapter Dirichlet kernels, Fejer kernels and Poisson kernels. Think a little about their different properties. I have the feeling that something very very deep is hidden in the different summability properties of the different kernels - but I do not understand what.

3 Problems to consider: Solve **3, 4, 5** and **1a** (on p. 65).

4. Assignment for the 4th of February:
Let $f(x)$ be a 2π periodic function such that $f(x) = \sum_{n=-\infty}^{\infty} c_n e^{inx}$ for some sequence c_n such that $\sum_{n=-\infty}^{\infty} |c_n| < \infty$. We want to find a $2\pi-$periodic solution, $y(x)$, to the following differential equation

$$y'(x) + ay(x) = f(x) \tag{1}$$

where $a \in \mathbb{R}$ and $a \neq 0$.

1. For any $N \in \mathbb{N}$ find a $2\pi-$periodic solution, $y_N(x)$, to

$$y_N'(x) + ay_N(x) = S_N(f)(x).$$

2. Carefully prove that there exists a $2\pi-$periodic function $y(x)$ such that $y_N(x) \to y(x)$ uniformly on $[0, 2\pi]$. That $y(x)$ is continuously differentiable and that $y(x)$ solves (1).

The point of the exercise is to practice proof writing. So you should make sure that every step of your deduction is carefully motivated. Often during the lectures and in the book results from standard analysis is not carefully explained in order to save space. You should explain those things.

You are to hand in a complete solution on the lecture on the 4th of February.

5 Office hours: I will have office hours in my office on level 7 in the mathematics building on Friday the 31st from 11-12pm in case you have any questions.