Assignments Week 2 SF2705 Fourieranalysis.

These are the things that you are expected to do before the Lecture on the **11th of February**.

1 Reading: Read the following in Stein-Shakarichi

- Chapter 2.4-2.5 pp 48-58 and Chapter 3.1 pp. 69-81

2 Discussion questions.

1. I do not really understand this myself. But you should reflect a little about why the Dirichlet kernel doesn't have the same good convergence properties as "good kernels". It has something to do with D_N not satisfying $D_N \geq 0$. There is something very very deep that the Fejer and Cesaro approximations converges point-wise for continuous functions. Whereas approximations by means of Dirichlet kernels does not converge point-wise.
2. What does Corollary 5.4, on page 54, imply for Fourier series and the applicability of Fourier theory to continuous functions.
3. Assuming that (which we will show soon) $S_N(f)(x) \nrightarrow f(x)$. Why does it make sense to introduce mean square convergence $\|S_N(f) - f\| \to 0$?

   Something very deep is going on here as well. We change the concept of convergence from point-wise convergence to convergence in a new way where we consider the function $f(x)$ as an object in a vector (Hilbert) space.
4. Is Corollary 2.4 still true if for C^2-functions on $[-\pi, \pi]$? Why, or why not?

3 Problems to consider: Solve **6, 9** and **11** in chapter 2 as well as problem **2** (on p. 66).

4. Assignments for the 11th of February:
Assignment 1: Let $f(x)$ be a $2\pi-$periodic function on $\mathbb{R}$. Assume furthermore that Hölder continuous:

$$\sup_{x \neq y} \frac{|f(x) - f(y)|}{|x - y|^\alpha} \leq C$$

for some constant C and $\alpha \in]0, 1]$.
Prove that

$$\hat{f}(n) = O(|n|^{-\alpha}).$$

Carefully explain every step of your proof.

Assignment 2: Explain in a few lines why it is intuitively reasonable that the Fourier coefficients $\hat{f}(n)$ tend to zero as $n \to \infty$ if f is a Hölder continuous function.

This exercise is less about proving a new and different version of Corollary 2.4. It is about reading mathematical theory, to see possibilities in proving new versions of theorems.
Furthermore, the proof of Corollary 2.4 in Stein-Shakarchi is not very instructive. It feels like a trick and does not indicate why smoothness (C^2 or any other assumption) should be related to the decay of the Fourier coefficients.

5 Office hours: I will have office hours in my office on level 7 in the mathematics building on Friday the 7th of February 10-11am in case you have any questions.