Assignments Week 5 SF2705 Fourieranalysis.

These are the things that you are expected to do before the Lecture on the **25th of February**.

1 Reading: We will finnish chapter 3 and try to summarize what we know about the convergence of Fourier series. The we will start looking at some applications in Chapter 4. You should therefore try to repeat chapter 1-3 and then read section 4.1-4.2 pp. 100-113 in Stein-Shakarichi

2 Discussion questions.

1. Does the Bolzano-Weierstrass theorem hold in an infinite dimensional Hilbert space. Does the bounded sequence $\{e^{inx}\}_{n=0}^{\infty}$ converge?
2. What does problem 6 tell you about the correspondence between integrable functions and the space l^2. Something of importance is at work here. We know that for every integrable function f we get an element in l^2 - but not the converse. Maybe that indicates that the space of integrable functions is not the right space to work in. Indeed, most commonly people prefer the space L^2 to do Fourier analysis on. In this course we will stick to the space of integrable functions.

3 Problems to consider: Solve **1, 2, 3, 6, 11** and **14** in chapter 3.

4. Assignments for the 25th of February:

Assignment 1: Let f be the continuous function with diverging partial Fourier sums $S_N(f)(0)$ defined on page 87 in Stein-Shakarchi. Let $f(x) \sim \sum_{n=-\infty}^{\infty} a_n e^{inx}$. Prove that there is a rearrangement of the sum $\sum_{n=-\infty}^{\infty} a_n e^{inx}$ that converges for $x = 0$. That is, prove that it is possible to change the order of summation to get the series to converge for $x = 0$.

Assignment 2:

1. **(Relies on some measure theory and will not be assessed.)** We know that if f is integrable on the circle then
$$\frac{1}{2\pi}\int_{-\pi}^{\pi} |f(x) - S_N(f)(x)|^2 \, dx \to 0 \quad \text{as } N \to \infty.$$
Prove that this implies that for any $\delta, \epsilon > 0$ there exists an $M_{\epsilon,\delta}$ such that the set
$$\{x \in (-\pi, \pi];\ |f(x) - S_N(f)(x)| > \epsilon\}$$
has has measure (see the appendix) less than δ for all $N > M_{\epsilon,\delta}$.
2. Explain why this does not imply that $S_N(f)(x) \to f(x)$ for all x outside a set of measure zero.

Hand in both assignments on the lecture on the 25th of February.

5 Office hours: I will have office hours in my office on level 7 in the mathematics building on Friday the 21st of February 10:30-11:30am in case you have any questions.