Assignments Week 8 SF2705 Fourieranalysis.

These are the things that you are expected to do before the Lecture on the **1st of April**.

1 Reading: Read chapter 5.1.1 to 5.1.5, pages 129-142.

2 Discussion questions.

1. We needed to define the space of functions of moderate decrease in order to be able to define the Fourier transform. Find a function (necessarily without moderate decrease) for which the Fourier transform is not defined.

2. We introduced the Fourier transform as an analogue to the Fourier coefficients. That indicate that we should be able to get analogue properties. For example, can we find an analogue for the Riemann-Lebesgue Lemma?

3. We have seen that the smoothness of a function $f(x)$, that is if $f \in C$, C^α, C^2 etc., provides decay estimates for the Fourier coefficients $\hat{f}(n)$ as $n \to \infty$. Is there an analogue for this for the Fourier transform? How, would you state it.

   In general if $f(x)$ is of moderate decrease we can not conclude that $\hat{f}(\xi)$ is of moderate decrease. But can we strengthen the assumptions on f to assure that $\hat{f}$ is of moderate decrease?

3 Problems to consider: This week I do not find any problems in the book that I find extremely interesting. So consider the following home made question instead:

Let $u(x,t)$, for $t \geq 0$, solve the heat equation $\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}$ on the circle for $t > 0$. Furthermore assume that $u(x,0) = f(x)$ where $f(x)$ is a given Riemann integrable function (see section 4.4 pp. 118-120 in S-S).

Prove that for any $t_0 > 0$ and any $k \in \mathbb{N}$ the partial derivatives $\frac{\partial^k u(x,t_0)}{\partial x^k}$ are continuous in x.

4. Assignment for the 1st of April: Let $f^j(x)$ be a sequence of integrable functions on the circle such that $\int_{-\pi}^{\pi} |f^j(x)|^2 dx = 1$. Furthermore let $u^j(x,t)$ solve the heat equation on the circle with initial data $u^j(x,0) = f^j(x)$.

Prove that for any $t_0 > 0$ there exists a function $u_{t_0}(x)$, and a subsequence u^{k_j}, such that

$$\int_{-\pi}^{\pi} \left| u^{k_j}(x,t_0) - u_{t_0}(x) \right|^2 dx \to 0 \qquad \text{as } j \to \infty.$$

Does it necessarily exist a function $f_0(x)$ such that

$$\int_{-\pi}^{\pi} |f^{k_j}(x) - f_0(x)|^2 dx \to 0?$$

5 Office hours: It does not seem to be any need for office hours. In case you have any pressing question please write me an email (johnan@kth.se) and we can book a time on Friday.