Homological algebra and algebraic topology
Problem set 7

due: Tuesday Nov 5 in class.

Problem 1 (3pt)**.** This exercise is a reading assignment, aimed at understanding the difference between reduced and unreduced cones and suspensions and, for an inclusion $i\colon A \hookrightarrow X$, the difference between its mapping cone C_i and the simple-minded quotient X/A. If i is well-behaved (a "cofibration"), these are homotopy equivalent. Read and understand the basics of cofibrations; you can choose from:

(1) Bredon, Chapter VII.1
(2) tom Dieck, Chapter 5.1 including Problems 5–8
(3) J.P. May, *A concise course in algebraic topology*, Chapter 6

or find your own reference.

To show you can work with these notions, give one example each (not from the literature) of

(1) a cofibration;
(2) a continuous inclusion $A \hookrightarrow X$ that is not a cofibration.

Give proofs, of course, but you don't need to reproduce what you've read.

Problem 2 (2pt)**.** Using the Eilenberg-Steenrod axioms, show the following for an arbitrary space X:

$$H_{n+1}(X \times \mathbf{S}^1) \cong H_n(X) \oplus H_{n+1}(X).$$

Problem 3 (2pt)**.** Computer the homology of the surface that is the boundary of a tubular neighborhood of the figure-eight graph in $\mathbf{R}^3$:

Problem 4 (3pt)**.** Use Brouwer's fixed point theorem to show that any square matrix all of whose entries are positive real numbers has a positive eigenvalue with a corresponding eigenvector consisting of non-negative entries.