Matematiska Institutionen
KTH

Homework number 1 to SF2736, fall 2013.

Please, deliver this homework at latest on Monday, November 18.

The homework must be delivered individually, and, in general, just handwritten notes are accepted. You are allowed to discuss the problems with your classmates, but you are not allowed to deliver a copy of the solution of another student.

1. (0.1p) The integer $x = 92$ solves the two congruences $x \equiv 12 (\text{mod } 20)$ and $x \equiv -4 (\text{mod } 24)$. Find all other solutions.

2. (0.2p) Find necessary and sufficient conditions for the integers a_1 and a_2 such that the two congruences $x \equiv a_1 (\text{mod } q_1)$ and $x \equiv a_2 (\text{mod } q_2)$ are simultaneously solvable.

3. Let p be a prime number. The elements in the direct product

$$Z_p{}^n = Z_p \times Z_p \times \cdots \times Z_p,$$

can be regarded as vectors with scalars in Z_p (instead of the real numbers as scalars). Defining addition of vectors and multiplication with scalars in the traditional way, Z_p^n becomes a vector space, that we denote by $V(n,p)$. These so called finite vector spaces are important in many applications used in real world. Linear independence, subspace and dimension are concepts that can be defined in the same way as they are defined in real vector space.

   (a) (0.1p) Find the number of 1-dimensional subspaces of $V(7,2)$.

   (b) (0.2p) Find the number of 2-dimensional subspaces of $V(n,p)$.

   (c) (0.2) Consider the space $V(4,5)$. Find the dimension of the kernel of the linear map represented by the matrix

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 1 & 2 \\ 0 & 1 & 3 & 2 \\ 2 & 4 & 3 & 3 \end{bmatrix}$$

   in the “standard” basis.

   (d) (0.2p) Find the number of 3×3-matrices $\mathbf{A}$ with elements in Z_p, such that the determinant of $\mathbf{A}$ is non-zero, (that is, p does not divide $\det(\mathbf{A})$).