

Matematiska Institutionen
KTH

Homework number 2 to SF2736, fall 2013.

Please, deliver this homework at latest on Monday, November 25.

Observe: You have to sign in for the exam on your “Mina Sidor” during the period November 25 to December 8. The solutions to the problems below, as well as any current information to the course, are published on the webpage <http://www.math.kth.se/math/GRU/2013.2014/SF2736/current%20information.html>.

1. (0.1) Let $A = \{1, 2, \dots, 7\}$ and let \mathcal{R} be the following relation on A :

$$\mathcal{R} = \{(1, 2), (2, 3), (3, 5), (7, 6), (7, 7)\}.$$

Find the size $|\mathcal{R}'|$ of the smallest equivalence relation \mathcal{R}' that contains \mathcal{R} .

2. (0.2p) Let \mathcal{M} be a set of size 7. Find the number of equivalence relations \mathcal{R} on \mathcal{M} such that $|\mathcal{R}| = 29$. (Some elementary combinatorics is needed for the solution.)
3. (0.2p) Are there any relations \mathcal{R} of size 15 on the set $\{1, 2, \dots, 6\}$ such that \mathcal{R} is both transitive and symmetric. (A solution with a correct answer but with an incorrect proof will give zero points.)
4. (0.1p) Show that if A is a countable infinite set of the real numbers then its complement is infinite but not countable infinite.
5. (0.1p) Is the family of equivalence relations on a countable infinite set always an uncountable infinite set?
6. (0.3p) Can you to any uncountable infinite set \mathcal{M} find an uncountable infinite family \mathcal{F} consisting of pairwise disjoint uncountable infinite subsets to \mathcal{M} ?