Matematiska Institutionen, KTH

Homework number 3 to SF2736, fall 2013.

Please, deliver this homework at latest on Monday, December 2.

The homework must be delivered individually, and, in general, just handwritten notes are accepted. You are free to discuss the problems below with your class mates, but you are not allowed to copy the solution of another student.

Observe: You have to sign in for the exam on your "Mina Sidor" during the period November 25 to December 8. The solutions to the problems below, as well as any current information to the course, are published on the webpage http://www.math.kth.se/math/GRU/2013.2014/SF2736/current%20information.html.

1. (0.2p) Let $\mathrm{H}(n,k)$ denote the number of ways to partition a set with n elements into k subsets of the same size. Derive a formula for $\mathrm{H}(n,k)$.

2. (a) (0.1p) Find a formula for the number of words of length $n+k$ you can form by using n a's and k b's and such that no two a's are adjacent.

   (b) (0.2p) Find the number of words of length 21 you can form by using eight a's, seven b's and six c's such that no two a's are adjacent.

3. Find the number of equivalence relations $\sim$ on the set $\{1, 2, 3, \ldots, 7\}$ such that

   (a) (0.1p) $1 \sim 2$ and $3 \sim 4$.

   (b) (0.2p) $1 \not\sim 2$, $1 \not\sim 3$ and $3 \not\sim 2$.

   Remark. The answers to the two problems above must, beside explanations, be given as an integer, that is, as an element in Z.

4. (0.2p) In the class we recently proved the following formula by deriving the number of positive integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 10$ in two distinct ways:

$$\binom{9}{3} = \binom{13}{3} - 4\binom{12}{2} + 6\binom{11}{1} - 4.$$

   Generalize this formula to a "new" equality for binomial coefficients.