**Problem session November 15, SF2736, fall 13. Please prepare!**

1. Is the following information sufficient to find the relation $\mathcal{R}$:
   1. The relation $\mathcal{R}$ is an equivalence relation on $\mathcal{M} = \{1, 2, 3, 4, 5, 6\}$.
   2. $\{(1, 2), (2, 3), (2, 4), (5, 6)\} \subseteq \mathcal{R}$.
   3. $(2, 6) \notin \mathcal{R}$.

2. Let $M = \{1, 2, 3, 4, 5, 6, 7\}$. Describe all equivalence relations $\mathcal{R}$ on $M$ such that
$$\{(1, 5), (1, 4), (2, 3), (3, 6)\} \in \mathcal{R}.$$

3. What is the mistake in the following proof for that a relation $\mathcal{R}$ which is symmetric and transitive must be reflexive: If $a\mathcal{R}b$ then by symmetry $b\mathcal{R}a$ and hence by transitivity $a\mathcal{R}b$ and $b\mathcal{R}a$ implies that $a\mathcal{R}a$

4. Assume that $f : A \to B$ and $g : B \to A$ are such that
$$(g \circ f)(x) = x \qquad \text{for all} \qquad x \in A\ .$$
Does this imply that $f$ and $g$, respectively, are either injective, surjective or bijective?

5. (a) Let $f : A \to B$, $g : B \to C$ and $h : C \to D$. Is it always true that for every $x \in A$
$$((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)\ .$$
   (b) Find two functions $f : A \to A$ and $g : A \to A$ such that
$$f \circ g \neq g \circ f\ .$$
   (c) Show that if $f : A \to B$ and $g : B \to C$ are bijective functions then the function $g \circ f : A \to C$ is a bijective function.

6. (a) Show that a union of any finite family of countable infinite sets is a countable infinite set.
   (b) Can the union of a countable infinite family of countable sets be countable infinite.
   (c) Is the set of bijective maps from $Z^+$ to $Z^+$ an infinite countable set?

7. Assume that $A$ is a given countable infinite set and let $B$ be the set of all real numbers $x$ that are solutions to some polynomial equation
$$a_0 + a_1x + a_2x^2 + \cdots + a_nx^n = 0,$$
where $a_i \in A$, for $i = 0, 1, \ldots, n$. Is the set $B$ countable infinite?

8. Let $S$ be a set of five positive integers the maximum of which is at most 9. Show that the sums of the elements in all the nonempty subsets of $S$ cannot all be distinct.

9. Show that to any sequence $a_1$, $a_2$, ..., $a_{p+1}$ of $p+1$ positive integers there exists at least one subsequence
$$a_{i_1},\ a_{i_2}, \ldots,\ a_{i_t}\ ,$$
such that
$$a_{i_1} + a_{i_2} + \ldots + a_{i_t} \equiv 0 \pmod{p}\ .$$