Problem session January 15, SF2736, winter 14.

1. Are there any family of subsets to the set $M = \{1, 2, 3, \ldots, 8\}$ such that every member of the family has size 5 and every element of M is contained in exactly three distinct subsets.

2. Find the number of integer solutions to the equation $x_1 + x_2 + x_3 + x_4 = 14$ such that $x_1 \geq 0$, $0 \leq x_2 \leq 3$, $0 \leq x_3 \leq 4$ and $0 \leq x_4 \leq 5$

3. A box contains white, red and green marbles, and pieces of papers with the numbers 1, 2 and 3. A sample of 12 is chosen with replacement, that is, every time an object is taken it is replaced in the box.
   (a) How many distinct sequences of selcetions can be found?
   (b) How many distinct samples can be found?

4. Show that every even permutation can be expressed as a product of 3-cycles.

5. Find the number of words of length n you can form using the letters A, B, C and D such that the number of A:s are even.

6. Show that if the elements a and b in a group satisfies $(ab)^2 = a^2b^2$ then $ab = ba$.

7. Let G be a group with the identity element e. Show that if $a^2 = e$ for every element a in G the G is an abelian group.

8. In how many ways can the corners of a tetraeder if at most m colors may be used.

9. A tree has 5 vertices of valency 2, four vertices of valency 3 and 2 vertices of valency 4. Find the number of verticies of valency 1.

10. Show that if a connected graph have just one vertex of valency 1 then the graph maust have at least one cycle.