

**Problem session December 2, SF2736, fall 13. Please prepare!**

1. Find an element  $x$  in the group  $\mathcal{S}_4$  of permutations on the set  $\{1, 2, 3, 4\}$  such that

$$(1\ 2\ 4\ 3)x(2\ 1\ 3\ 4) = (1\ 4)(2\ 3).$$

2. Show that there are no elements  $\varphi$  in the group  $\mathcal{S}_5$  of permutations on the set  $\{1, 2, 3, 4, 5\}$  such that

$$\varphi^2 = (1\ 2\ 3)(2\ 3\ 4\ 5).$$

3. Show that the following multiplication table is not the multiplication table of a group:

$\circ$	$e$	$a$	$b$	$c$	$d$
$e$	$e$	$a$	$b$	$c$	$d$
$a$	$a$	$b$	$d$	$e$	$c$
$b$	$b$	$e$	$c$	$d$	$a$
$c$	$c$	$d$	$a$	$b$	$e$
$d$	$d$	$c$	$e$	$a$	$b$

4. Can the following table be completed to the multiplication table of a group.

$\circ$	$e$	$a$	$b$	$c$	$d$
$e$	$e$				
$a$		$e$			
$b$					
$c$					
$d$					

5. (a) Find the smallest subgroup of  $(\mathbb{Z}_{18}, +)$  that contains the elements 3 and 7.  
 (b) Find the smallest coset of some subgroup of  $(\mathbb{Z}_{18}, +)$  that contains the elements 3 and 7.
6. (a) For any two subgroups  $H$  and  $K$  of a group  $G$  show that  $H \cap K$  is a subgroup of  $G$ .  
 (b) Show that there does not exist a group  $G$  with two subgroups  $H$  and  $K$ , such that neither  $H \subseteq K$  nor  $K \subseteq H$ , and such that  $H \cup K$  is a subgroup of  $G$ .  
 (c) The sizes of the subgroups  $H$  and  $K$  of  $G$  are 52 and 151, respectively, find the size of  $H \cap K$ .
7. Find a non abelian group of size 66.
8. Is the group  $(\mathbb{Z}_{19} \setminus \{0\}, \cdot)$  a cyclic group.
9. Let  $\mathcal{S}_4$  denote the group consisting of all permutations on the set  $\{1, 2, 3, 4\}$ . Find the smallest subgroup of  $\mathcal{S}_4$  that contains the permutations  $(1\ 2\ 3)$  and  $(3\ 4)$ .
10. Show that every subgroup of a cyclic group is cyclic.
11. Show that every group with 55 elements contains at least one element of order 5 and at least one element of order 11.
12. Find a group with 64 elements, of which all have order either 1 or 2.