

**Problem session December 9, SF2736, fall 13.**

**Please prepare!**

1. In how many ways can the faces of a tetrahedron be colored in  $q$  distinct colors.
2. The matrix

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$

is the parity-check matrix of an 1-error-correcting code  $C$ .

- (a) Find all elements of  $C$ .
  - (b) Correct the word 011111.
  - (c) How many words cannot be corrected.
3. Find a linear 1-error-correcting code  $C$  of length 9, size  $|C| = 32$  and containing the words 111100000 and 110000110.
  4. For which integers  $n$  are there an 1-error-correcting binary code  $C$  that can correct all words of length  $n$ .
  5. Find the maximum size of a 2-error-correcting code of length 8.
  6. Show that

$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \dots + n\binom{n}{n} = 2^{n-1}n.$$

7. Solve the recursion

$$u_{n+2} + 8u_{n+1} - 9u_n = 8 \cdot 3^{n+1}, \quad n = 2, 3, \dots$$

and where  $u_0 = 2$  and  $u_1 = -6$ .

8. Find the number of partitions of 16 in which each part is an odd prime.
9. Prove that the number of partitions of  $n$  in which each part is 1 or 2 is equal to the number of partitions of  $n + 3$  which have exactly two distinct parts.