1.

$$\tilde{f}(x) = \pi/4 + \sum_{n=1}^{\infty} \left(\frac{1-(-1)^n}{n^2 \pi} \cos(nx) + \frac{1}{n} \sin(nx) \right)$$

$$\tilde{f}(0) = \pi/2$$ (compare chapter 11.2, p. 429)

2.

$$u(x,y) = A_0 x + \sum_{n=1}^{\infty} A_n \sinh\left(\frac{n\pi}{b}x\right) \cos\left(\frac{n\pi}{b}y\right)$$

$$A_0 = \frac{1}{ab}\int_0^b g(y)\,dy \,, \quad A_n = \frac{2}{b \sinh \frac{n\pi}{b}a} \int^b g(y) \cos\left(\frac{n\pi}{b}y\right) dy$$

(compare chapter 12^0, p. 475)

3.

$$\begin{vmatrix} 3-\lambda & 1 \\ 1 & 3-\lambda \end{vmatrix} = 8 + \lambda^2 - 6\lambda = (\lambda-2)(\lambda-4)$$

$$\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\Rightarrow \vec{x}_H(t) = c_+ e^{4t}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + c_- e^{2t}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$\vec{x}_p = \begin{pmatrix} u \\ v \end{pmatrix}$$ (Ansatz) = constant (vector)

$$\Rightarrow \vec{x}_p = -\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}^{-1}\begin{pmatrix} 2 \\ 4 \end{pmatrix} = -\frac{1}{8}\begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 2 \\ 4 \end{pmatrix} = -\frac{1}{4}\begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

$$\vec{x}(t) = \vec{x}_H(t) + \vec{x}_p(t)$$