1. $\dot{x} = x^2 + y^2 - 6 \overset{!}{=} 0$

$\dot{y} = x^2 - y \overset{!}{=} 0$ $\quad \Longrightarrow_{(y=x^2)} \quad y^2 + y - 6 \overset{!}{=} 0$

$y = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + 6} \overset{!}{\geq} 0$

$\vec{X}_1 = \begin{pmatrix} \sqrt{2} \\ 2 \end{pmatrix} \quad \vec{X}_2 = \begin{pmatrix} -\sqrt{2} \\ 2 \end{pmatrix} \qquad \Rightarrow y = 2, \quad x_\pm = \pm\sqrt{2}$

$J(x,y) = \begin{pmatrix} 2x & 2y \\ 2x & -1 \end{pmatrix} \Rightarrow J_i = \begin{pmatrix} \pm 2\sqrt{2} & 4 \\ \pm 2\sqrt{2} & -1 \end{pmatrix},$

$|J_i - \lambda \mathbb{1}| = \lambda^2 + \lambda(1 \mp 2\sqrt{2}) \mp 10\sqrt{2} \overset{!}{=} 0$

$\Rightarrow$ Egenvärden av J_1 : $\lambda_\pm = \left(\sqrt{2} - \frac{1}{2}\right) \pm \sqrt{\left(\sqrt{2} - \frac{1}{2}\right)^2 + 10\sqrt{2}} \gtrless 0$

J_2 : $\lambda'_\pm = -\left(\sqrt{2} + \frac{1}{2}\right) \pm \sqrt{\underbrace{\left(\sqrt{2} + \frac{1}{2}\right) - 10\sqrt{2}}_{= -\omega^2}}$

$= -\alpha^2 \pm i\omega$

$\Rightarrow \left(\vec{X}(t) - \vec{X}_1\right) = e^{\lambda_+ t}\vec{V}_+ + e^{\lambda_- t}\vec{V}_-, \quad \lambda_+ > 0 \Rightarrow$ unstable

$\left(\vec{X}(t) - \vec{X}_2\right) = e^{-\alpha^2 t}\left(e^{i\omega t}\vec{V}_+ + e^{-i\omega t}\vec{V}_-\right) \to \vec{0} \quad (t \to +\infty)$

dvs 'stable'!

2. $y(t) = \begin{cases} 5e^{-t}, & 0 \leq t < \pi \\ 5e^{-t} + \frac{3}{2}\left(e^{-(t-\pi)} + \sin t + \cos t\right), & t \geq \pi \end{cases}$

(cf. Example 9, p. 296)

3. $\hat{X}(\omega) = 5 + 7e^{-i\omega}\, \mathcal{FT}(\cos t) + 3e^{+i\omega}\, \mathcal{FT}\left(\frac{1}{t^2+1}\right)$

$= 5 + 7e^{-i\omega}\pi\left(\delta(\omega+1) + \delta(\omega-1)\right) + 3e^{i\omega}\pi e^{-|\omega|}$