Institutionen för matematik
KTH
Chaotic Dynamical Systems, Fall 2014
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Homework assignment 2

This exercise set is due October 17, 2014

1. (Devaney p. 93, 2) Let $E_\lambda(x) = \lambda e^x$. Determine the phase portrait for E_λ for $\lambda > 0$.
2. (Devaney p. 93, 3) Consider E_λ as in the previous excercise but for $\lambda < 0$. Prove that E_λ has a unique fixed point that is attracting for $0 > \lambda > -e$ and is repelling for $\lambda < -e$.
3. (Devaney p. 93, 4)
   a) Prove that E_λ^2 is convex for x such that $E'_\lambda(x) > -1$ and concave if $E'_\lambda(x) < -1$.
   b) Consider $E_\lambda(x)$ for $x < 0$. Use the information from a) to prove that E_λ has a unique attracting fixed point if $-e < \lambda < 0$, and that for $\lambda < -e$ it has a repelling fixed point coexisting with a stable two-orbit.
4. Define a metric on $\Sigma_N = \{\mathbf{s} = (s_k)_{k=0}^{\infty} \mid s_k \in 1, \dots, N\}$ by

$$d_N(\mathbf{s}, \mathbf{t}) = \sum_{k=0}^{\infty} \frac{|s_k - t_k|}{N^k}.$$

   a) Prove that d_N is a metric.
   b) Prove that if $s_i = t_i$ for $i = 0, \dots, k$ then $d_N(\mathbf{s}, \mathbf{t}) \leq 1/N^k$. Similarly if $d_N(\mathbf{s}, \mathbf{t}) < 1/N^k$, then $s_i = t_i$ for $i \leq k$.

   For symbols s and t in the symbol space $\{1, 2, \dots, N\}$, define the discrete metric by

$$\delta(s,t) = \begin{cases} 0 \text{ if } s = t \\ 1 \text{ if } s \neq t. \end{cases}$$

   Define a metric d' on Σ_N by

$$d'(\mathbf{s}, \mathbf{t}) = \sum_{k=0}^{\infty} \frac{\delta(s_i, t_i)}{3^i}$$

c) Prove that the metrics d_N and d' are *not equivalent*, i.e. that there *do not exist constants* c and C so that

$$c\, d_N(\mathbf{s}, \mathbf{t}) \leq d'(\mathbf{s}, \mathbf{t}) \leq C d_N(\mathbf{s}, \mathbf{t}),$$

where c and C do not depend on $\mathbf{s}$ and $\mathbf{t}$.

Does one of the relations between the metrics stated above hold?

d) State and prove a version of the statement in b) for the metric d'.

5. (Devaney p. 68, 5) Construct a piecewise linear map with period $2n + 1$.
6. (Devaney p. 68, 8) Construct a map that has periodic points of period 2^j for $j < l$ but no points of period 2^l.