Chaotic dynamical systems, 2014
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Computer Exercise 1

The purpose of this exercise is to study the quadratic family

$$x \mapsto 1 - ax^2, \qquad 0 < a \leq 2.$$

1. The bifurcation diagram for $f_x(x) = 1 - ax^2$. The task is to draw the diagram which describes how the dynamical properties of the quadratic family varies as you vary the parameter a. You may use a method of your choice (computer type and language) but here follows a description of one possibility:

For $0 < a < 1/\sqrt{2}$, f_a has a unique fixed point which becomes a stable two period which is superstable when $a = 1$. As is proven in the course the following holds:

If f_a has a stable periodic orbit this orbit attracts the critical point 0.

To decide whether for a given parameter value a, f_a has a stable periodic orbit we therefore iterate the point 0 300 times to make the sequence approach the periodic orbit that possibly exists and then we plot the next 300 points. A c-program, `quad.c` that computes the succesive plots is available at `http://www.math.kth.se/~michaelb/quad.c`. The program is compiled on a unix system by the command `cc quad.c -o quad`. To execute the program with output on the file `res` you should give the command `quad > res`. The obtained result can then be plotted by for instance the program `gnuplot`. You can also save the plotted result in Postscript format to be printed later. If you prefer you can use a high level language like Matlab.

2. *Feigenbaums period doublings I.* Your task is to determine the sequence $\{a_j\}_{j=1}^{\infty}$ of parameter values for which f_a has a superstable 2^j cycle. Then estimate the Feigenbaum constant δ which is defined by the formula

$$\delta = \lim_{n\to\infty} \frac{a_n - a_{n-1}}{a_{n+1} - a_n}.$$

3∗. Feigenbaums period doublings II. Try to determine as many as possible of the points $\{b_n\}_{n=1}^{\infty}$ where the bifurcations from a stable 2^n period to a stable 2^{n+1} period takes place. These points are given as solutions $a = b_n$ to the system of equations

$$\begin{cases} f_a^{2^n}(x) = x \\ Df_a^{2^n}(x) = -1. \end{cases}$$

Then give an estimate of the Feigenbaum constant δ by the formula

$$\delta = \lim_{n\to\infty} \frac{b_n - b_{n-1}}{b_{n+1} - b_n}.$$

You may use and computer tools and programming languages for 2. and 3∗. you like. Possible choices can be Matlab, Maple, Mathematica or a traditional computer language.