Homework Assignment 1, Selected topics in PDE, Fall 2014

September 30, 2014

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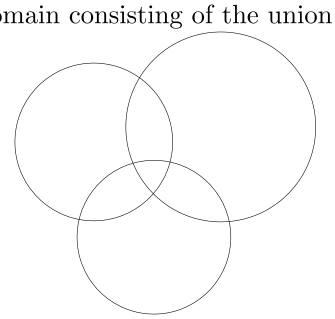
In the file under the link *"Chapters 3"* on the course page:

HTTP://WWW.MATH.KTH.SE/MATH/GRU/2014.2015/SF2723/

we begin by sketching an algorithm on how to construct a solution to the Dirichlet problem in a simply connected domain consisting of the union of three balls.



In this homework you are supposed to prove the existence for the Dirichlet problem in such a domain.

Problem formulation: *Let $B_{r_1}(x^1)$, $B_{r_2}(x^2)$ and $B_{r_3}(x^3)$ be three given balls in $\mathbb{R}^n$, say $n \geq 3$, and $D = \cup_{i=1}^3 B_{r_i}(x^i)$ is a simply connected domain as in the figure. Furthermore, let $f(x) \in C(\partial D)$ be a given continuous function. Prove that the Dirichlet problem*

$$\begin{aligned} \Delta u(x) &= 0 \qquad \text{in } D \\ u(x) &= f(x) \quad \text{on } \partial D \end{aligned}$$

admits a bounded solution. You only need to verify that the boundary data $u(x) = f(x)$ is satisfied at regular points of ∂D.[1]

Instructions: You may use any method of proof that you want that is correct - creativity is not dissuaded. However, you may not refer to the Perron's method or any other already established method. To be specific, if your answer is something like *"This is a direct application of Theorem X"* then you have not shown that you understand the PDE theory that we have developed. The point of the homework is to force you to engage the theory we have been developing and to show that you understand it. I would suggest that you structure your proof as follows:

1. Reduce the problem to the case when $f(x) \geq 0$.
2. Use the strategy outlined in the beginning of *"Chapters 3"* to construct a sequence v^k.
3. Prove that v^k is increasing and bounded from above and thus convergent to some function v^0.
4. Prove that $\Delta v^0(x) = 0$.
5. Use Theorem 3 in "Chapters 2" to show that $v^0(x) \to f(x)$ for every $x \in \partial D \setminus \{x;\ x \in \partial B_{r_i}(x^i) \cap B_{r_j}(x^j),\ i \neq j\}$.

You may use any other method that is correct as long as you show that you understand the theory that we have been developing.

Your solutions, together with this sheet, should be handed in during the lecture on the 21st October.

Good Luck!

[1] You may thus disregard the three singular cusps in ∂D.