SF2729 Groups and Rings
Problem set 1

due: Tuesday Nov 11 in class

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class, put them in the homework box outside the *studentexpedition* in the math department or send them by email to `tilmanb@kth.se`. If you scan homework and send it by email, send it as a single PDF file with high contrast (no photos please). I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Let d be a non-negative integer and put
$$P = \{(x, y) \in \mathbf{Z} \times \mathbf{Z} \mid x^2 - dy^2 = 1\}.$$
Let (x_0, y_0) and (x_1, y_1) be elements in P. Consider the following operation:
$$(x_0, y_0) \cdot (x_1, y_1) = (x_0 x_1 + d y_0 y_1, x_0 y_1 + x_1 y_0).$$
Show that $\cdot$ is a binary operation on P and that P endowed with this operation is a group.

Problem 2. Complete the following table so that it is the multiplication table of a group. There is only one possibility. You do not need to write down your reasoning in this exercise. *Warning:* Pay attention to associativity!

$\cdot$	e	p	q	r	s	t
e	e	p	q	r	s	t
p	p	q	e	s		
q	q					
r	r			e		p
s	s					
t	t					

Problem 3. Show that the set $\mathrm{SL}_n(\mathbf{Z})$ of n-by-n matrices with integer entries and determinant one is a group under matrix multiplication.