SF2729 Groups and Rings
Problem set 10

due: Friday February 20 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `wojtek@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Show that the ring $\mathbb{Z}[\sqrt{-2}]$ is a Euclidean domain. Conclude that all the ideals in $\mathbb{Z}[\sqrt{-2}]$ are generated by one element (this ring is PID). Compute a generator of the ideal generated by 3 and $-1 + 5\sqrt{-2}$.

Problem 2. Show that $\mathbb{Z}[i]/(2 + 5i)$ is a finite field. How many elements does it have?

Problem 3. Consider the ring $R = \mathbb{Z}/10$. Show that the ideal generated by 5 is prime. Conclude that $S = R \setminus (5)$ is a multiplicative set. Describe the ring $R[S^{-1}]$.