SF2729 Groups and Rings
Problem set 11

due: Friday February 27 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `wojtek@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Let p be a prime number with $p = 1(\text{mod } 4)$. Recall that this implies existence of a and b in $\mathbb{Z}$ such that $p = a^2 + b^2$ (as we shown during the last exercises). Prove that a and b are unique up to order and signs.

Problem 2. Let R be an integral domain and $p(x) = a_n x^n + \quad + a_1 x + a_0$ be an irreducible polynomial in $R[x]$. Show that then also the polynomial $q(x) = a_0 x^n + \quad + a_{n-1} x + a_n$ is irreducible.

Problem 3. Show that the rings $\mathbb{Z}[i]/29$ and $\big(\mathbb{Z}[i]/(2+5i)\big) \times \big(\mathbb{Z}[i]/(2-5i)\big)$ are isomorphic. Tip: Chinese Remainder theorem.