SF2729 Groups and Rings

Problem set 12

due: Friday March 6 via e-mail or in the homework box at the expedition of the mathematics department.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `wojtek@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Let R be a UFD. For a polynomial $f = a_0 + a_1 x + \cdots + a_n x^n$ in $R[x]$, define $c(f) := \gcd\{a_0, a_1, \ldots, a_n\}$. Prove that $c(fg) = c(f)c(g)$.

Problem 2. Prove that $\mathbb{Z}[2\sqrt{2}][x]$ is not a UFD.

Problem 3. Show that $(x-1)(x-2)(x-3)(x-4) - 1$ is irreducible in $\mathbb{Q}[x]$.