

SF2729 Groups and Rings

Problem set 2

due: Thursday Nov 20 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Consider the symmetric group S_8 of permutations of $\{1, \dots, 8\}$.

- (1) Compute the product $\sigma = (14286)(352)(1784)(34)$ in cycle notation. (Remember that we apply cycles from right to left.)
- (2) What is the order of σ ?
- (3) Is there an element in S_8 which has a larger order than σ ?

Problem 2. Let G be the set of all real 2×2 -matrices of the form $\begin{pmatrix} a & b \\ b & a \end{pmatrix}$ with $a \neq \pm b$.

- (1) Show that G is an abelian group under matrix multiplication.
- (2) Show that if $A \in G$ has finite order, then it has order 1 or 2. (*Hint:* all elements of G are diagonalizable.)

Problem 3. Compute the order $|G| \in \mathbb{N} \cup \{\infty\}$ of the group G with the presentation

$$G = \langle x, y \mid x^2y = 1, xy^3 = 1 \rangle.$$