# SF2729 Groups and Rings
# Problem set 4

due: Wednesday Dec 3 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

**Problem 1.** Show that the group

$$(\mathbf{Z}/9\mathbf{Z})^{\times} = \{\bar{a} \in \mathbf{Z}/9\mathbf{Z} \mid \gcd(a, 9) = 1\}$$

is cyclic. Which elements of this group are generators?

**Problem 2.** Let $H < G$ be a subgroup and

$$\overline{H} = \bigcap \left\{ K \trianglelefteq G \mid H \subseteq K \right\}$$

Show that (a) $\overline{H}$ is a normal subgroup of $G$, and (b) that it is the smallest normal subgroup of $G$ containing $H$ in the following sense: if $N$ is any normal subgroup of $G$ containing $H$ then $\overline{H} \leq N$.

**Problem 3.** Show that $\mathrm{SL}_n(\mathbf{C})$ is a normal subgroup of $\mathrm{GL}_n(\mathbf{C})$ and that the quotient group $\mathrm{GL}_n(\mathbf{C})/\mathrm{SL}_n(\mathbf{C})$ is isomorphic to $\mathbf{C}^{\times}$.