SF2729 Groups and Rings
Problem set 5

due: Thursday Dec 11 in class.

As there was no class this week, please read the rest of Chapter 3.1, then Chapter 1.7 and 3.2 in the book. The problems this week are about this new material. You'll have the opportunity to ask questions about the homework or the reading next Wednesday; the homework itself is due on Thursday. Both classes next week will be a mix of lecture and exercise session.

Problem 1. Let G be any group and define G' to be the subgroup generated by commutators, i. e. elements of the form

$$[x, y] := xyx^{-1}y^{-1}; \quad x, \ y \in G.$$

Show that G' is a normal subgroup of G and that G/G' is abelian.

Problem 2. Let G be the full symmetry group of a cube (with reflections, it has order 48). Compute the stabilizers of

    (1) one of the eight corners;
    (2) the midpoint of one of the twelve edges;
    (3) the midpoint of one the the six faces.

Problem 3. Show that every proper subgroup of a group G of order 91 is cyclic. Show that G is either abelian or $Z(G) = \{1\}$.