

# SF2729 Groups and Rings

## Problem set 6

due: Thursday Dec 18 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `tilmanb@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

**Problem 1.** A group  $G$  is called *metabelian* if it has a normal abelian subgroup  $A \trianglelefteq G$  such that  $G/A$  is also abelian. Show that any subgroup of a metabelian group is metabelian. (*Hint:* use the Second Isomorphism Theorem.)

**Problem 2.** Let  $G = \mathrm{SL}_2(\mathbf{R})$  denote the group of  $2 \times 2$ -matrices with determinant 1 and let  $\mathbf{H} = \{z \in \mathbf{C} \mid \Im(z) > 0\}$  be the upper half-plane of complex numbers with positive imaginary part. Show that  $G$  acts on  $\mathbf{H}$  by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \cdot z := \frac{az + b}{cz + d}.$$

Compute the kernel of this action and the stabilizer of  $z = i$ .

**Problem 3.** Read about conjugacy in symmetric groups, p. 125–126. Determine the sizes of all conjugacy classes of  $S_5$  (given in the table at the top of p. 127) and verify that the class equation holds.

(Do not write a list of all the 120 elements. Instead, give a short argument why the sizes of the classes are what you claim.)