SF2729 Groups and Rings
Problem set 8

due: Friday February 6 in class.

Write clear, clean, brief, and complete solutions and use whole sentences. Solutions without proper reasoning score worse. You can submit hand-written or typed solutions and turn them in in class or send them by email to `wojtek@kth.se`. I will not accept late homework except under extraordinary circumstances that you need to discuss with me before the deadline.

Problem 1. Denote by R the set of all polynomials in $f(x) \in \mathbb{Q}[x]$ with the property that $f(n) \in \mathbb{Z}$ for all $n \in \mathbb{Z}$. Show that R is a unital subring of $\mathbb{Q}[x]$ and give an example of a polynomial in R which is not in $\mathbb{Z}[x] \subset \mathbb{Q}[x]$.

Problem 2. Let R be a commutative ring with 1. Let I be the ideal in $R[X]$ generated by $X^2 - 1$. Show that the group ring $R[\mathbb{Z}/2]$ and the quotient ring $R[X]/I$ are isomorphic.

Problem 3. Given a homomorphism of commutative rings $f\colon R \to S$, show that:

(1) If I is an ideal in R, then $f(I)$ is an ideal in $f(R)$ but not necessarily in all of S. (Give a counterexample for the latter statement.)
(2) If J is an ideal in S, the $f^{-1}(J)$ is an ideal in R.