Matematiska Institutionen
KTH

**Exam to the course Discrete Mathematics, SF2736, April 9, 2015, 08.00-13.00.**

**Observe:**

1. Nothing else than pencils, rubber, rulers and papers may be used.
2. Bonus marks from the homeworks will be added to the sum of marks on part I. The maximum number of marks on part I is 15.
3. Grade limits: 13-14 points will give Fx; 15-17 points will give E; 18-21 points will give D; 22-27 points will give C; 28-31 points will give B; 32-36 points will give A.
4. **Observe.** All answers must be justified with a complete argumentation!!

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# Part I

1. (a) (1p) Find $535^{176} \pmod{89}$.

   (b) (2p) Solve in the ring $Z_{53}$ the matrix equation

$$\begin{pmatrix} 10 & 12 \\ 13 & 14 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

2. (3p) In how many ways can 10 boys and 13 girls be placed in a row in such a way that no two boys are adjacent.

3. Let $G$ denote the group which is the direct product $G = (Z_4, +) \times (Z_6, +)$.

   (a) (1p) Find and describe a cyclic subgroup $H$ to $G$ of size $|H| = 12$.

   (b) (1p) Is there any cyclic subgroup $K$ to $G$ of size $|K| = 8$.

   (c) (1p) Find the number of subgroups to $G$ of size 2.

4. (3p) Solve, by using the technique with generating functions, the recursion

$$a_n = a_{n-1} + 12a_{n-2}, \qquad n = 2, 3, \ldots$$

   where $a_0 = 2$ and $a_1 = 1$

5. (a) (1p) Is there any connected graph $G$, without multiple edges and loops, with 11 vertices of the degrees (valencies) 1, 2, 3, 4, ..., 11, respectively.

   (b) (2p) Find the minimum number of edges that must be deleted in the complete graph $K_n$ so that the graph that remains will not be connected.

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# Part II

6. (a) (1p) Give a formula for the number of words with $n$ letters of which $m$ are identical and the remaining letters distinct.

   (b) (2p) Prove the identity.

$$\sum_{i=0}^{k} \binom{n+i}{i} = \binom{n+k+1}{k}.$$

7. (4p) Find the minimum and maximum number of edges in a bipartite graph with 53 vertices admitting an Euler circuit.

8. (4p) Let $G$ denote group of (multiplicatively) invertible elements in the ring $Z_{32}$. For which integers $n$ is there a group of permutations $\mathcal{S}_n$ of the elements in the set $\{1, 2, \ldots, n\}$ such that $\mathcal{S}_n$ contains a subgroup isomorphic with $G$.

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# Part III

9. Let $D$ be the smallest linear code containing the words 11111111, 10011001 and 11100001.

   (a) (1p) Show that $D$ is a 1-error-correcting code.

   (b) (2p) Find two distinct 1-error-correcting codes $C_1$ and $C_2$ distinct from $D$ and containing $D$. (1p for each such code.)

   (c) (3p) Give a non-trivial upper bound for the number of linear 1-error-correcting codes $C$ that contain $D$.

10. (4p) Is the set of all equivalence relations on the set of positive integers an uncountable or a countable infinite set? Justify your answer with good motivations.