Matematiska Institutionen
KTH

Homework number 1 to SF2736, fall 2014.

Please, deliver this homework at latest on Monday, November 17, 2014. Provide both your name and your e-mail address with your solutions.

The homework must be delivered individually, and, in general, just hand-written notes are accepted. You are allowed to discuss the problems with your classmates, but you are not allowed to deliver a copy of the solution of another student.

1. (0.1p) Find $700^{1734} (\mathrm{mod}\ 347)$.

2. (0.2p) Find all solutions to the Diophantine equation

$$346y + 512z = 10.$$

3. (0.3p) Let p be a prime number less than or equal to 13, and let a and b be elements in the ring $\mathbb{Z}_p$. Find the number of solutions in $\mathbb{Z}_p$ to the system of equations

$$\begin{cases} x + y + z = 1 \\ x + 2y + (a+1)z = b+1 \\ x + 3y + (a^2+2a+2)z = 3b+1 \end{cases}$$

4. (0.4) For which integer sequences a_1, a_2, ..., a_t is it true that

$$\gcd(a_1, a_2, \ldots, a_t)\,\mathrm{lcm}(a_1, a_2, \ldots, a_t) = a_1 a_2 \cdots a_t\,.$$