Matematiska Institutionen
KTH

Homework number 2 to SF2736, fall 2014.

Please, deliver this homework at latest on Monday, November 24, 2014. Provide both your name and your e-mail address with your solutions.

The homework must be delivered individually, and, in general, just hand-written notes are accepted. You are allowed to discuss the problems with your classmates, but you are not allowed to deliver a copy of the solution of another student.

1. (0.1p) Let $M = \{1, 2, \ldots, 9\}$. Find the smallest equivalence relation $\mathcal{R}$ such that $\mathcal{R}$ contains the set

$$\mathcal{A} = \{(1,2),(5,2),(3,3),(4,9),(5,7),(7,7),(9,3)\}.$$

2. (0.1p) Let M, $\mathcal{A}$ and $\mathcal{R}$ be as above, Is there any subset $\mathcal{B}$ of $M \times M$ such that $|\mathcal{B}| < |\mathcal{A}|$ and such that $\mathcal{R}$ is the smallest equivalence relation containing $\mathcal{B}$.

3. (0.2p) Is there any equivalence relation $\mathcal{R}$ of size 21 (i.e., $|\mathcal{R}| = 21$) on some set M such that $\mathcal{R}$ induces a partition of M into four equivalence classes.

4. (0.1p) Find countable infinite sets A and B and a function f from A to B such that if a function g from B to A is such that $g \circ f$ is the identity on A then $f \circ g$ must be the identity on B.

5. Let A and B be countable infinite sets. Are there any functions f from A to B such that the number of functions g from B to A with the property that $g \circ f$ is the identity on A, while $f \circ g$ is not the identity on B, is

   (a) (0.1p) finite,

   (b) (0.2p) countable infinite,

   (c) (0.2p) uncountable infinite.