Matematiska Institutionen
KTH

Homework number 5 to SF2736, fall 2014.

Please, deliver this homework at latest on Monday, December 15, 2014. Provide both your name and your e-mail address with your solutions.

The homework must be delivered individually, and, in general, just hand-written notes are accepted. You are allowed to discuss the problems with your classmates, but you are not allowed to deliver a copy of the solution of another student.

1. (0.1p) Find the number of words that cannot be corrected by the 1-error-correcting code C defined by the parity-check matrix

$$\begin{pmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix}$$

   Give examples of words that does not belong to C but can be corrected to a word in C.

2. (0.2p) Find the number of distinct necklaces that you can produce with ten beans that are either black or white. The answer must be given as an integer.

3. (0.3p) Solve, by using the technique with generating functions, the recursion

$$a_n - a_{n-1} - 12a_{n-2} = n, \qquad n = 2, 3, \ldots,$$

   where $a_0 = 1$ and $a_1 = 2$.

4. (0.4p) In the direct product $\mathbb{Z}_3^n = \mathbb{Z}_3 \times \cdots \times \mathbb{Z}_3$ of n copies of the ring $\mathbb{Z}_3$ we define the distance between two vectors, or words, $\bar{x} = x_1 \ldots x_n$ and $\bar{y} = y_1 \ldots y_n$ by

$$\mathrm{d}(\bar{x}, \bar{y}) = |\{i \mid x_i \neq y_i\}|.$$

   Generalize the concept of linear 1-error-correcting binary codes to codes consisting of words of length n formed by using the elements in $\mathbb{Z}_3$ as "letters". Construct a linear 1-error-correcting code with as many words as possible in the direct product $\mathbb{Z}_3^7 = \mathbb{Z}_3 \times \cdots \times \mathbb{Z}_3$.