Problem session November 10, SF2736, fall 14.

Please prepare!

0. Find the inverse of the element 76 in the ring Z_{221}.

1. Solve

   (a) $13x + 18 = 13$ in the ring Z_{64}.

   (b)
$$\begin{cases} 7x + 2y = 5 \\ 10x + 7y = 3 \end{cases}$$
   in the ring Z_{13}.

2. Find all elements b of Z_{17} such that the equation
$$x^2 + 3x + b = 0 ,$$
   has a solution.

3. Find
$$545^{112} \pmod{23} \qquad \text{and} \qquad 545^{112} \pmod{24} .$$

4. Show that if p is a prime number then
$$(p-1)! \equiv -1 \pmod{p} .$$

5. Find all integers n and m such that $314n + 218m = 12$ with $0 \leq n \leq 20$.

6. Show that for every positive integer n there is an integer m, and digits a_i, with $0 \leq a_i \leq 9$, for $i \in \{n, n+1, \ldots, k\}$, such that
$$347^m = (a_k a_{k-1} a_{k-2} \ldots a_n 00 \ldots 01)_{10}$$

7. For any two integers a and b show that the two integers $a/\gcd(a,b)$ and $b/\gcd(a,b)$ are relatively prime.

8. Find the number of solutions of the equation $x^2 = 1$ in the ring Z_{990}.

9. Solve the equation $(x-5)(x+3) = 0$ in the ring Z_{56}.

10. (a) Assume that m, n and k are integers with $n \geq 2$ and $m \geq 2$, and assume that $n^2 = km^2$. Prove that k is a square of some integer.

    (b) Assume that $\gcd(x, y) = 1$. Prove that if $xy = z^2$ for some integer z, then $x = n^2$ and $y = m^2$ for some integers n and m.

11. Show that if $2^n - 1$ is a prime number then n must be a prime number. Will the same statement be true if we substitute 2 by any integer $a \geq 2$.

12. (a) Find the number of solutions to an equation
$$a_1x_1 + a_2x_2 + \ldots + a_nx_n = b ,$$
    in a ring Z_p, where p is a prime number.

    (b) Give a more general result from which your answer to the previous problem follows.