Problem session December 8, SF2736, fall 14.

Please prepare!

1. In how many ways can the faces of a tetrahedron be colored in q distinct colors.

2. The matrix
$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix}$$
   is the parity-check matrix of an 1-error-correcting code C.

   (a) Find all elements of C.

   (b) Correct the word 011111.

   (c) How many words cannot be corrected.

3. Find a linear 1-error-correcting code C of length 9, size $|C| = 32$ and containing the words 111100000 and 110000110.

4. For which integers n are there an 1-error-correcting binary code C that can correct all words of length n.

5. Find the maximum size of a 2-error-correcting code of length 8.

6. Show that
$$\binom{n}{1} + 2\binom{n}{2} + 3\binom{n}{3} + \ldots + n\binom{n}{n} = 2^{n-1}n \ .$$

7. Solve the recursion
$$u_{n+2} + 8u_{n+1} - 9u_n = 8 \cdot 3^{n+1}, \qquad n = 2, 3, \ldots$$
   and where $u_0 = 2$ and $u_1 = -6$.

8. Find the number of partitions of 16 in which each part is an odd prime.

9. Prove that the number of partitions of n in which each part is 1 or 2 is equal to the number of partitions of $n + 3$ which have exactly two distinct parts.