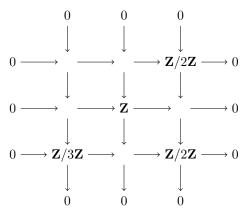
Homological algebra and algebraic topology Problem set 2

due: Tuesday Sept 14 in class.

Problem 1 (3pt). Assume that the following diagram of abelian groups has exact rows and columns. Can you determine the missing entries and maps? Give short reasonings.



Problem 2 (4pt). Let C_{\bullet} be a chain complex and denote by $C_{\bullet}/2C_{\bullet}$ the chain complex which in degree *n* is the quotient groups $C_n/2C_n$. Consider the sequence

$$0 \to C_{\bullet} \xrightarrow{2} C_{\bullet} \to C_{\bullet}/2C_{\bullet} \to 0.$$

- (1) Show that this sequence is exact if all C_n are free abelian groups.
- (2) Give a (preferably small) example of such a C_{\bullet} that has a nontrivial connecting homomorphism $\delta_n \colon H_{n+1}(C_{\bullet}/2C_{\bullet}) \to H_n(C_{\bullet})$.

Problem 3 (3pt). Let C be a category. Denote by $\operatorname{Ar}(C)$ the following category: an object in $\operatorname{Ar}(C)$ is a morphism $X_1 \to X_2$ in C. A morphism in $\operatorname{Ar}(C)$ from $X_1 \to X_2$ to $Y_1 \to Y_2$ is a commutative diagram

$$\begin{array}{c} X_1 \longrightarrow Y_1 \\ \downarrow & \qquad \downarrow \\ X_2 \longrightarrow Y_2 \end{array}$$

- (1) Show that $\operatorname{Ar}(\mathcal{C})$ is indeed a category.
- (2) Show that $F: \operatorname{Ar}(\operatorname{Ab}) \to \operatorname{Ar}(\operatorname{Ab})$ with $F(X_1 \xrightarrow{f} X_2) = (\ker(f) \hookrightarrow X_1)$ defines a functor.