Homological algebra and algebraic topology
Problem set 6

due: Monday Oct 19.

Problem 1 (3pt). Let $R = \mathbf{Z}/8\mathbf{Z}$ and $M = R/(4) \cong \mathbf{Z}/4\mathbf{Z}$. Compute $\mathrm{Tor}_n^R(M, M)$.

Problem 2 (2pt+2pt). Let k be a field, $R = k[x_1, x_2]$, and $M = R/(x_1, x_2) \cong k$. Compute $\mathrm{Ext}_R^n(M, M)$.

Bonus: compute the same for $R = k[x_1, \dots, x_n]$, $M = R/(x_1, \dots, x_n)$.

Problem 3 (3pt). Show that the following spaces are homotopy equivalent:

(1) $\mathbf{D}^n = \{x \in \mathbf{R}^n \mid |x| \leq 1\}$ and a point;
(2) $\mathbf{R}^n - \{0\}$ and $\mathbf{S}^{n-1} = \{x \in \mathbf{R}^n \mid |x| = 1\}$;
(3) The complement of $\mathbf{S}^1 = \{x \in \mathbf{R}^4 \mid |x| = 1, x_3 = x_4 = 0\} \subset \mathbf{S}^3$ and $\mathbf{S}^1$.

Problem 4 (2pt). Show that if $f,\ g\colon X \to Y$ are homotopic maps then their mapping cones C_f, C_g are homotopy equivalent.