

16) Prove that parallel transport preserves lengths, and calculate the angle between a tangent vector at the north pole N of S^2 , and the vector obtained by parallel transport along the meridian γ to the equator, then along the equator and finally back to N along a meridian $\tilde{\gamma}$. What angle θ with N .

17) Show that for an arc-length parametrized geodesic $\vec{c}(s) = \vec{c}(u(s), v(s))$ on a surface of revolution $\vec{r} = (f(u)\cos v, f(u)\sin v, h(u))$

$$\ddot{u} - \frac{f' f''}{f^2} v^2 + \frac{f' f'' + h' h''}{f^2 + h'^2} \dot{v}^2 = 0$$

follows from the second geodesic equation

$$\ddot{v} + 2 \frac{f' f''}{f} \dot{u} \dot{v} = 0$$

18) By explicitly calculating the left-hand side and comparing with the Gauss consistency equations derived earlier in index notation prove that if X, Y and Z are tangent vector fields along a hypersurface $S \subset \mathbb{R}^N$,

$$\nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X, Y]} Z = \chi(Z, Y) \langle X, Y \rangle - \chi(Z, X) \langle Y, X \rangle.$$