FÖRSLAG TILL LÖSNING TENTAMEN I MATEMATIK BASKURS (SF1612) OCH MATEMATIK I FÖR CL (SF1623 TEN1) 2008-10-10

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1. $9+\sqrt{2x-3}=x$, $2x-3\geq 0 \iff x\geq \frac{3}{2}$

$\sqrt{2x-3}=x-9$

$(\sqrt{2x-3})^2=(x-9)^2$

$2x-3=x^2-18x+81$

$x^2-20x+84=0$

$x=10\pm\sqrt{100-84}$

$x=10\pm\sqrt{16}$

$x=10\pm 4$

$x=14$

$x=6$

PRÖVN AV RÖTTER: $x=14$ $VL=9+\sqrt{28-3}=9+\sqrt{25}=9+5=14=HL$

$x=6$ $VL=9+\sqrt{12-3}=9+\sqrt{9}=9+3=12\neq HL$

SVAR: $x=14$

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2. $6x^3-x^2-5x+2=0$ $\qquad P(x)=6x^3-x^2-5x+2$

PRÖVN. $P(1)=6\cdot 1^3-1^2-5+2=8-6=2\neq 0$

$P(-1)=6\cdot(-1)^3-(-1)^2-5(-1)+2=-6-1+5+2=0$

$\therefore x=-1$ ÄR EN ROT. DIVIDERA $P(x)$ MED $x+1$

$$\begin{array}{l|l}
6x^3-x^2-5x+2 & x+1 \\
-(6x^3+6x^2) & 6x^2-7x+2 \\
\quad -7x^2-5x & \\
\quad -(-7x^2-7x) & \\
\qquad 2x+2 & \\
\qquad 2x+2 & \\
\qquad\quad 0 &
\end{array}$$

$\therefore 6x^3-x^2-5x+2=$

$=(6x^2-7x+2)(x+1)$

$(6x^2-7x+2)(x+1)=0$

$6x^2-7x+2=0$

$x^2-\frac{7}{6}x+\frac{2}{6}=0$

$x=\frac{7}{12}\pm\sqrt{\frac{49}{144}-\frac{48}{144}}$

$x=\frac{7}{12}\pm\sqrt{\frac{1}{144}}$

$x=\frac{7}{12}\pm\frac{1}{12}$

$x=\frac{8}{12}=\frac{2}{3}$

$x=\frac{1}{2}$

SVAR: RÖTTER ÄR

$$\begin{cases} x=\frac{1}{2} \\ x=\frac{2}{3} \\ x=-1 \end{cases}$$

3.

$$\frac{1}{2}\cos x + \sin^2 x = 1$$
$$\frac{1}{2}\cos x + (1-\cos^2 x) = 1$$
$$\frac{1}{2}\cos x + 1 - \cos^2 x - 1 = 0$$
$$\frac{1}{2}\cos x - \cos^2 x = 0$$
$$\cos x\left(\frac{1}{2} - \cos x\right) = 0$$

1) $\cos x = 0$
$$x = \frac{\pi}{2} + n\cdot\pi$$

2) $\frac{1}{2} - \cos x = 0$
$$\cos x = \frac{1}{2}$$
$$x = \pm\frac{\pi}{3} + n\cdot 2\pi$$

SVAR: $x = \frac{\pi}{2} + n\cdot\pi$, $x = \pm\frac{\pi}{3} + n\cdot 2\pi$

4.

$$3 + \frac{4}{x} \le x$$
$$0 \le x - \frac{4}{x} - 3$$
$$0 \le \frac{x^2 - 4 - 3x}{x}$$
$$0 \le \frac{(x-4)(x+1)}{x}$$

$$\frac{(x-4)(x+1)}{x} \geq 0$$

x		-1		0		4	
$x-4$	$-$		$-$		$-$	0	$+$
$x+1$	$-$	0	$+$		$+$		$+$
x	$-$		$-$	0	$+$		$+$
$\frac{(x-4)(x+1)}{x}$	$-$	0	$+$	$\nexists$	$-$	0	$+$

$-1 \le x < 0$ $x \geq 4$

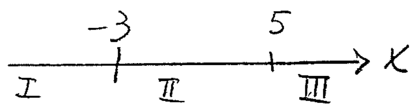
SVAR: $-1 \le x < 0$ ELLER $x \geq 4$

5.

$$|x+3| + |x-5| = 8$$

$$|x+3| = \begin{cases} x+3 & \text{OM } x+3 \geq 0 \\ -(x+3) & \text{OM } x+3 < 0 \end{cases}$$

$$|x-5| = \begin{cases} x-5 & \text{OM } x-5 \geq 0 \\ -(x-5) & \text{OM } x-5 < 0 \end{cases}$$



I: $x < -3$; $-(x+3) + (-(x-5)) = 8$
$$-x - 3 - x + 5 = 8$$
$$-2x = 8 - 2$$
$$x = -3 \qquad x \notin I$$

II: $-3 \le x < 5$; $x + 3 - (x-5) = 8$
$$8 = 8$$
IDENTITET!

III: $x \geq 5$
$$x + 3 + x - 5 = 8$$
$$2x = 8 + 2$$
$$x = 5 \in III$$

SVAR: $-3 \le x \le 5$

6.

$$2\ln(x-1)+\ln x-\ln(x^2-1)=0$$

DEF:
$x-1>0 \Leftrightarrow x>1$
$x>0$
$x^2-1>0 \Leftrightarrow x>1,\ x<-1$ $\Big\} \Rightarrow x>1$

$$\ln(x-1)^2+\ln x-\ln(x^2-1)=\ln 1$$

$$\ln\frac{(x-1)^2\cdot x}{x^2-1}=\ln 1$$

$$\frac{(x-1)^2\cdot x}{x^2-1}=1$$

$$\frac{(x-1)(x-1)\cdot x}{(x-1)(x+1)}=1$$

$$\frac{(x-1)\cdot x}{x+1}=1$$

$$x^2-x=x+1$$

$$x^2-2x-1=0$$

$$x=1\pm\sqrt{1+1}$$

$$x=1\pm\sqrt{2}$$

$x>1$ GER $x=1+\sqrt{2}$.

SVAR: $x=1+\sqrt{2}$.

7a) VISA $\sin 3x=3\sin x-4\sin^3 x$

$$\sin 3x=\sin(x+2x)=\big|\sin(a+b)=\sin a\cos b+\cos a\sin b\big|$$

$$=\sin x\cos 2x+\cos x\sin 2x=\sin x(1-2\sin^2 x)$$

$$+\cos x\cdot 2\sin x\cos x=\sin x-2\sin^3 x+2\sin x\cos^2 x=$$

$$=\sin x-2\sin^3 x+2\sin x(1-\sin^2 x)=$$

$$=\sin x-2\sin^3 x+2\sin x-2\sin^3 x=3\sin x-4\sin^3 x$$

V.S.V.

7b) $\sin 3x=2\sin x$

$$3\sin x-4\sin^3 x=2\sin x$$

$$\sin x-4\sin^3 x=0$$

$$\sin x(1-4\sin^2 x)=0$$

1) $\sin x=0$

$x=n\cdot\pi$

2) $1-4\sin^2 x=0$

$\sin^2 x=\frac{1}{4}$

$\sin x=\pm\frac{1}{2}$

$\sin x=\frac{1}{2}$

$x=\frac{\pi}{6}+n\cdot 2\pi$

$x=\frac{5\pi}{6}+n\cdot 2\pi$

$\sin x=-\frac{1}{2}$

$x=-\frac{\pi}{6}+n\cdot 2\pi$

$x=-\frac{5\pi}{6}+n\cdot 2\pi$

SVAR: $x=n\cdot\pi$, $x=\pm\frac{\pi}{6}+n\cdot 2\pi$

$x=\pm\frac{5\pi}{6}+n\cdot 2\pi$

8. VISA $4^n - 3n - 1$ DELBART MED 9, $n \geq 2$

$P(n)$: $4^n - 3n - 1 = 9k$, k HELA TAL

$P(2)$: $4^2 - 3 \cdot 2 - 1 = 16 - 6 - 1 = 9 = 9 \cdot 1$ DVS DELBART MED 9.

ANTAG $P(n)$ SANN, VISA $P(n+1)$

$P(n+1)$: $4^{n+1} - 3(n+1) - 1 = 4^n \cdot 4 - 3n - 3 - 1 =$

$= \left| 4^n = 9k + 3n + 1 \right| = (9k + 3n + 1) \cdot 4 - 3n - 4 =$

$= 36k + 12n + 4 - 3n - 4 = 36k + 9n = 9(4k + n)$

!! DELBART MED 9.

DVS $P(n) \Rightarrow P(n+1)$

SLUTSATS: $4^n - 3n - 1$ DELBART MED 9, $n \geq 2$ ENLIGT INDUKTIONSPRINCIPEN.

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9. VISA $\binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}$

$$\binom{n}{k} = \frac{n!}{(n-k)!\,k!} \qquad \binom{n}{k+1} = \frac{n!}{(n-k-1)!\,(k+1)!}$$

$$\binom{n+1}{k+1} = \frac{(n+1)!}{(n+1-k-1)!\,(k+1)!} = \frac{(n+1)!}{(n-k)!\,(k+1)!}$$

$$VL = \binom{n}{k} + \binom{n}{k+1} = \frac{n!}{(n-k)!\,k!} + \frac{n!}{(n-k-1)!\,(k+1)!} =$$

$$= \frac{n!}{(n-k)(n-k-1)!\,k!} + \frac{n!}{(n-k-1)!\,(k+1)k!} = \left| \text{MGN} = (n-k)!\,(k+1)! \right|$$

$$= \frac{n! \cdot (k+1) + n!\,(n-k)}{(n-k)!\,(k+1)!} = \frac{k \cdot n! + n! + n \cdot n! - k \cdot n!}{(n-k)!\,(k+1)!} =$$

$$= \frac{n! + n \cdot n!}{(n-k)!\,(k+1)!} = \frac{n!\,(n+1)}{(n-k)!\,(k+1)!} = \frac{(n+1)!}{(n-k)!\,(k+1)!} = \binom{n+1}{k+1} = HL$$

V.S.V.