1.

$$\frac{1}{1-x} \geq \frac{1}{1+x} \qquad x \neq \pm 1$$

$$\frac{1}{1-x} - \frac{1}{1+x} \geq 0$$

$$\frac{1+x-(1-x)}{(1-x)(1+x)} \geq 0$$

$$\frac{2x}{(1-x)(1+x)} \geq 0$$

		-1		0		1	
$2x$	$-$		$-$	0	$+$		$+$
$1-x$	$+$		$+$		$+$	0	$-$
$1+x$	$-$	0	$+$		$+$		$+$
$\frac{2x}{(1-x)(1+x)}$	$+$	$\nexists$	$-$	0	$+$	$\nexists$	$-$

$x < -1$ ELLER $0 \leq x < 1$

SVAR! $x < -1$ ELLER $0 \leq x < 1$

2.

$$\left(2x - \frac{1}{4x^2}\right)^7 = \sum_{k=0}^{7} \binom{7}{k} (2x)^{7-k} \left(-\frac{1}{4x^2}\right)^k$$

$$\left.\begin{aligned} (2x)^{7-k} &= 2^{7-k} \cdot x^{7-k} \\ \left(-\frac{1}{4x^2}\right)^k &= (-1)^k \cdot \left(\frac{1}{4}\right)^k \cdot (x^{-2})^k \end{aligned}\right\} \quad \begin{aligned} x^{7-k} \cdot (x^{-2k}) &= x^{7-3k} \equiv x^1 \\ 7-3k &= 1 \\ 6 = 3k &\Leftrightarrow k = 2 \end{aligned}$$

$$\binom{7}{2}(2x)^5\left(-\frac{1}{4x^2}\right)^2 = \left| \binom{7}{2} = \frac{7!}{5!2!} = \frac{7 \cdot 6}{2} = 21 \right| = 21 \cdot 2^5 \cdot x^5 \cdot \frac{1}{4^2} \cdot \frac{1}{x^4} =$$

$$= 21 \cdot 32 \cdot x^5 \cdot \frac{1}{16} \cdot \frac{1}{x^4} = 21 \cdot 2 \cdot x = 42x$$

∴ KOEFF. ÄR 42.

SVAR! 42

3.

$$2\sin x = \tan 2x$$

$$2\sin x = \frac{\sin 2x}{\cos 2x}$$

$$2\sin x \cdot \cos 2x = \sin 2x$$

$$2\sin x \cdot \cos 2x - \sin 2x = 0$$

$$2\sin x \cdot \cos 2x - 2\sin x\cos x = 0$$

$$2\sin x(\cos 2x - \cos x) = 0$$

1) $2\sin x = 0$

$$\sin x = 0$$
$$x = n \cdot \pi$$

2) $\cos 2x - \cos x = 0$

$$\cos 2x = \cos x$$
$$2x = \pm x + n \cdot 2\pi$$

i) $2x = x + n \cdot 2\pi$

$$x = n \cdot 2\pi$$

ii) $2x = -x + n \cdot 2\pi$

$$3x = n \cdot 2\pi$$
$$x = n \cdot \frac{2\pi}{3}$$

TILLSAMMANS 1) o 2)

$x = n \cdot \pi$ ELLER $x = n \cdot \frac{2\pi}{3}$

SVAR! $x = n \cdot \pi$ ELLER $x = n \cdot \frac{2\pi}{3}$, n HELTAL

4. $4^{-x}\cdot e^{x^2} = 2^{-\ln 2}$

$(2^2)^{-x}\cdot e^{x^2} = 2^{-\ln 2}$

$2^{-2x}\cdot e^{x^2} = 2^{-\ln 2}$

$e^{x^2} = \dfrac{2^{-\ln 2}}{2^{-2x}}$

$e^{x^2} = 2^{2x-\ln 2}$

$\ln e^{x^2} = \ln\left(2^{2x-\ln 2}\right)$

$x^2 = (2x-\ln 2)\ln 2$

$x^2 - 2x\ln 2 + \ln 2\cdot\ln 2 = 0$

$x = \ln 2 \pm \sqrt{(\ln 2)^2-(\ln 2)^2}$

$x = \ln 2.$

SVAR! $x = \ln 2$

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5. $\left|\dfrac{2x+3}{x-4}\right| = 5\ ,\ x\neq 4$

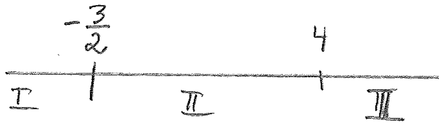
$\dfrac{|2x+3|}{|x-4|} = 5$

$|2x+3| = 5|x-4|$

$|2x+3| - 5|x-4| = 0$

$|2x+3| = \begin{cases} 2x+3 & \text{DÅ } 2x+3\geq 0 \\ -(2x+3) & \phantom{\text{DÅ }} 2x+3<0 \end{cases}$

$|x-4| = \begin{cases} x-4 & x-4>0 \\ -(x-4) & x-4<0 \end{cases}$



I: $x < -\frac{3}{2}$

$-(2x+3) - 5(-(x-4)) = 0$

$-2x-3+5x-20 = 0$

$3x-23=0$

$x = \frac{23}{3}$ TILLHÖR EJ I

II: $-\frac{3}{2} \leq x < 4$

$2x+3 - 5(-(x-4)) = 0$

$2x+3+5x-20 = 0$

$7x-17=0$

$x = \frac{17}{7} = 2\frac{3}{7}$ TILLHÖR II

III: $x > 4$

$2x+3-5(x-4) = 0$

$2x+3-5x+20=0$

$-3x+23=0$

$x = \frac{23}{3} = 7\frac{2}{3}$ TILLHÖR III

SVAR: $x = 2\frac{3}{7}$ ELLER $x = 7\frac{2}{3}$

OBS! ALT. LÖSNING! $\dfrac{2x+3}{x-4} = \pm 5 \quad x\neq 4$

1) $\dfrac{2x+3}{x-4} = 5$ GER $x = 7\frac{2}{3}$ 2) $\dfrac{2x+3}{x-4} = -5$ GER $x = 2\frac{3}{7}$

6.

$y = 4\sin\left(\frac{x}{3} + \frac{\pi}{4}\right) \quad 0 \le x \le 6\pi$

$y = \sin x$ HAR SITT STÖRSTA VÄRDE $= 1$ DÅ $x = \frac{\pi}{2} + n \cdot 2\pi$

$y = 4\sin\left(\frac{x}{3} + \frac{\pi}{4}\right)$ HAR SITT STÖRSTA VÄRDE DÅ

$\frac{x}{3} + \frac{\pi}{4} = \frac{\pi}{2} + n \cdot 2\pi \iff x = \frac{3\pi}{4} + n \cdot 6\pi$

DÅ $n = 0$ BLIR $x = \frac{3\pi}{4}$ OCH $0 \le x \le 6\pi$.

$y = \sin x$ HAR SITT MINSTA VÄRDE $= -1$ DÅ $x = \frac{3\pi}{2} + n \cdot 2\pi$

$y = 4\sin\left(\frac{x}{3} + \frac{\pi}{4}\right)$ HAR SITT MINSTA VÄRDE DÅ

$\frac{x}{3} + \frac{\pi}{4} = \frac{3\pi}{2} + n \cdot 2\pi \iff x = \frac{15\pi}{4} + n \cdot 6\pi$

$x = \frac{15\pi}{4}$ LIGGER I INT. $0 \le x \le 6\pi$.

STÖRSTA VÄRDE ÄR $4 \cdot 1 = 4$, MINSTA VÄRDE ÄR $4 \cdot (-1) = -4$.

<u>SVAR!</u> STÖRSTA VÄRDE ÄR 4, DÅ $x = \frac{3\pi}{4}$
MINSTA VÄRDE ÄR -4, DÅ $x = \frac{15\pi}{4}$

7.

$x^3 + px^2 - px - q = 0 \quad x = 1 + \sqrt[3]{2}$.

$x^2 = (1 + \sqrt[3]{2})^2 = 1 + 2\sqrt[3]{2} + \sqrt[3]{2} \cdot \sqrt[3]{2} = 1 + 2\sqrt[3]{2} + \sqrt[3]{4}$

$x^3 = (1 + \sqrt[3]{2})^3 = (1 + \sqrt[3]{2})(1 + \sqrt[3]{2})^2 = (1 + \sqrt[3]{2})(1 + 2\sqrt[3]{2} + \sqrt[3]{4})$

$= 1 + 2\sqrt[3]{2} + \sqrt[3]{4} + \sqrt[3]{2} + 2\sqrt[3]{4} + \sqrt[3]{8} = 1 + 3\sqrt[3]{2} + 3\sqrt[3]{4} + 2$

$= 3 + 3\sqrt[3]{2} + 3\sqrt[3]{4}$.

$x^3 + px^2 - px - q = 0$ BLIR MED INSATT x, x^2, x^3

$3 + 3\sqrt[3]{2} + 3\sqrt[3]{4} + p(1 + 2\sqrt[3]{2} + \sqrt[3]{4}) - p(1 + \sqrt[3]{2}) - q = 0$

$3 + p - p - q + \sqrt[3]{2}(3 + 2p - p) + \sqrt[3]{4}(3 + p) = 0$

$3 - q + \sqrt[3]{2}(3 + p) + \sqrt[3]{4}(3 + p) = 0$

$\therefore \quad 3 - q = 0 \iff q = 3$

$3 + p = 0 \iff p = -3$ <u>SVAR!</u> $p = -3$ OCH $q = 3$

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8. $\ln(x(\sqrt{1+e^x}+1)) + \ln(\sqrt{1+e^x}-1) = x + \ln x \quad , \quad x > 0$

$$\ln x + \ln(\sqrt{1+e^x}+1) + \ln(\sqrt{1+e^x}-1) = x + \ln x$$

$$\ln x + \ln(\sqrt{1+e^x}+1)(\sqrt{1+e^x}-1) = x + \ln x$$

$$\ln x + \ln(1+e^x-1) = x + \ln x$$

$$\ln x + \ln e^x = x + \ln x$$

$$\ln x + x = x + \ln x \quad \therefore \text{ ALLTID SANT.}$$

SVAR! EKVATIONEN GÄLLER FÖR ALLA x DÅ $x > 0$

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9. $y = \dfrac{x^2+4}{2x}$, $x \neq 0$ VILKA VÄRDEN KAN Y INTE ANTA?

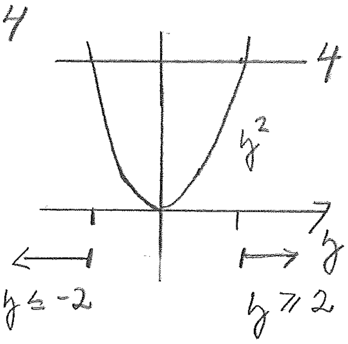
$$y \cdot 2x = x^2+4$$

$$x^2 - 2xy + 4 = 0$$

$$x = y \pm \sqrt{y^2-4}$$

$$y^2 - 4 \geq 0$$

$$y^2 \geq 4$$



$\therefore \quad \dfrac{x^2+4}{2x} \leq -2$ ELLER $\dfrac{x^2+4}{2x} \geq 2.$

VÄRDEN MELLAN -2 OCH 2 ANTAS EJ.

SVAR! $-2 < y < 2.$