FÖRSLAG TILL LÖSNING TENTAMEN I MATEMATIK BASKURS SF1612, SF1623. 2009-10-05

1\. a) $P(x) = x^3 - x - 6$

$$\frac{x^3 - x - 6}{x-2} = x^2 + 2x + 3$$

$$
\begin{array}{rrrr|l}
x^2 & +2x & +3 & & \\
\hline
x^3 & & -x & -6 & x-2 \\
-x^3 & \overset{+}{(-)}2x^2 & & & \\
\hline
 & 2x^2 & -x & & \\
 & -2x^2 & \overset{+}{(-)}4x & & \\
\hline
 & & 3x & -6 & \\
 & & -3x & \overset{+}{(-)}6 & \\
\hline
 & & & 0 &
\end{array}
$$

SVAR: $x^2 + 2x + 3$

b) OM $x = 2$ ÄR EN ROT TILL $x^3 - x - 6 = 0$ SÅ ÄR $x - 2$ EN FAKTOR I $x^3 - x - 6$ DVS $x^3 - x - 6 = (x-2) \cdot q(x)$ FÖR NÅGOT POLYNOM $q(x)$. (FAKTOR-SATSEN)

$P(2) = 2^3 - 2 - 6 = 8 - 2 - 6 = 0$ $\therefore$ $x = 2$ ÄR EN ROT. $\therefore$ $x - 2$ ÄR EN FAKTOR I $x^3 - x - 6 \Leftrightarrow x^3 - x - 6$ JÄMNT DELBART MED $x - 2$.

2\. a)

$$\ln x + \ln 3 = \ln(x+3)$$

$$\ln 3x = \ln(x+3)$$

$$3x = x + 3$$

$$2x = 3$$

$$x = \frac{3}{2}$$

SVAR $x = \frac{3}{2}$

b)

$$\frac{2^{3x+4} - 16}{2^{6x} - 2^{3x}} = 4$$

$$\frac{2^{3x} \cdot 2^4 - 2^4}{2^{3x} \cdot 2^{3x} - 2^{3x}} = 2^2$$

$$\frac{2^4 (2^{3x} - 1)}{2^{3x} (2^{3x} - 1)} = 2^2$$

$$\frac{2^4}{2^{3x}} = 2^2$$

$$2^{4-3x} = 2^2$$

$$4 - 3x = 2$$

$$-3x = 2 - 4$$

$$3x = 2$$

$$x = \frac{2}{3}$$

SVAR: $x = \frac{2}{3}$

3. $\cos^2 x = 3\sin x - 3$

$1 - \sin^2 x - 3\sin x + 3 = 0$

$\sin^2 x + 3\sin x - 4 = 0$ SÄTT $\sin x = t$

$t^2 + 3t - 4 = 0$

$t = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{16}{4}} = -\frac{3}{2} \pm \sqrt{\frac{25}{4}}$

$t = -\frac{3}{2} \pm \frac{5}{2}$ $\quad t_1 = 1$, $t_2 = -4$

1) $\sin x = 1$

$x = \frac{\pi}{2} + n \cdot 2\pi$

2) $\sin x = -4$

EJ DEF. $|\sin x| \le 1$

SVAR! $x = \frac{\pi}{2} + n \cdot 2\pi$

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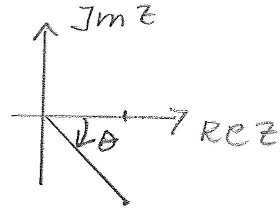
4. $\left(\frac{1}{z} - \frac{1}{\bar{z}} + 4\right)^5$ $\quad z = \frac{1+i}{4}$ $\quad \bar{z} = \frac{1-i}{4}$

$I = \frac{1}{z} - \frac{1}{\bar{z}} + 4 = \frac{1}{\frac{1+i}{4}} - \frac{1}{\frac{1-i}{4}} + 4 = \frac{4}{1+i} - \frac{4}{1-i} + 4 =$

$= \frac{4(1-i) - 4(1+i) + 4(1+i)(1-i)}{(1+i)(1-i)} = \frac{4 - 4i - 4 - 4i + 4(1-i^2)}{1 - i^2}$

$= \frac{-8i + 4(1+1)}{1+1} = \frac{8 - 8i}{2} = 4 - 4i = 4(1-i)$

$I^5 = (4(1-i))^5 = 4^5 (1-i)^5$



$1 - i = \sqrt{2}\, e^{-\frac{\pi}{4} i}$ $\quad (1-i)^5 = \left(\sqrt{2}\, e^{-\frac{\pi}{4} i}\right)^5$

$= 2^{\frac{5}{2}} \cdot e^{-\frac{5\pi}{4} i} = 2^{\frac{5}{2}}\left(\cos\left(\frac{5\pi}{4}\right) - i \sin\left(\frac{5\pi}{4}\right)\right) = 2^{\frac{5}{2}}\left(\left(-\frac{1}{\sqrt{2}}\right) + i \frac{1}{\sqrt{2}}\right)$

$= 2^2(-1 + i)$

$I^5 = 4^5 \cdot 2^2(-1+i) = 2^{10} \cdot 2^2 (-1+i) = 2^{12}(-1+i)$

SVAR: $2^{12}(-1+i)$

5.
$$\left(2x^2-\frac{3}{x}\right)^{11} = \sum_{k=0}^{11}\binom{11}{k}(2x^2)^k\left(-\frac{3}{x}\right)^{11-k} = \sum_{k=0}^{11}\binom{11}{k}2^k x^{2k}\cdot\frac{(-3)^{11-k}}{x^{11-k}}$$

$$= \sum_{k=0}^{11}\binom{11}{k}2^k\cdot(-3)^{11-k}\,x^{2k-(11-k)}$$

x^{10} INTRÄFFAR DÅ $2k-11+k=10$
$3k=21 \iff k=7$

$\therefore$ KOEFF. FRAMFÖR x^{10} ÄR $\binom{11}{7}\cdot 2^7\cdot(-3)^{11-7} =$

$$= \frac{11!}{7!4!}\cdot 2^7\cdot(-3)^4 = \frac{11\cdot 10\cdot \cancel{9}^{3}\cdot \cancel{8}^{4}}{4\cdot 3\cdot 2\cdot 1}\cdot 2^7\cdot 3^4 = 11\cdot 10\cdot 3\cdot 2^7\cdot 3^4$$

$= 110\cdot 3\cdot 2^7\cdot 3^4 = 110\cdot 2^7\cdot 3^5$ SVAR: $110\cdot 2^7\cdot 3^5$
(3421440)

6. $\ln(x+5) - \ln 2 + \ln x = \ln[(x+2)^2-6]$

D_f: $x+5>0$, $x>0$, $(x+2)^2-6>0 \iff x<-2-\sqrt{6}$ ELLER $x>-2+\sqrt{6}\approx 0{,}4$
TILLSAMMANS D_f: $x>-2+\sqrt{6}$

$$\ln\left(\frac{x(x+5)}{2}\right) = \ln[(x+2)^2-6]$$

$$\frac{x(x+5)}{2} = (x+2)^2-6$$

$$x^2+5x = 2(x^2+4x+4-6)$$

$$x^2-2x^2+5x-8x+4=0$$

$$-x^2-3x+4=0$$

$$x^2+3x-4=0$$

$$x = -\frac{3}{2}\pm\sqrt{\frac{9}{4}+\frac{16}{4}} \qquad x = -\frac{3}{2}\pm\frac{5}{2} \qquad \begin{matrix} x=1 \\ x=-4 \end{matrix}$$

$x=1$ SAT., $x=-4$ ÄR EJ ROT TY $(-4+2)^2-6<0$

SVAR: $x=1$

7.

$$\frac{x+3}{x-1} > x \iff 0 > x - \left(\frac{x+3}{x-1}\right) \iff 0 > \frac{x(x-1)-(x+3)}{x-1}$$

$$\frac{x^2-x-x-3}{x-1} < 0 \iff \frac{x^2-2x-3}{x-1} < 0 \iff \frac{(x+1)(x-3)}{x-1} < 0$$

x		-1		1		3	
$x-1$	$-$		$-$	0	$+$		$+$
$x+1$	$-$	0	$+$		$+$		$+$
$x-3$	$-$		$-$		$-$	0	$+$
$\frac{(x+1)(x-3)}{x-1}$	$-$	0	$+$	$\nexists$	$-$	0	$+$

$\frac{(x+1)(x-3)}{x-1} < 0$ DÅ

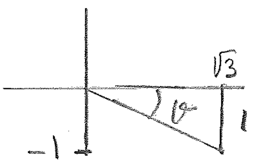
$x < -1$ ELLER $1 < x < 3$

SVAR! $x < -1$ ELLER $1 < x < 3$

8.

$$f(x) = \sqrt{3}\sin x - \cos x = \left| \sin(x+y) = \sin x \cos y + \cos x \sin y \right| = *$$

$$\begin{cases} \sqrt{3} = r\cos y \\ -1 = r\sin y \end{cases}$$



$\tan v = \frac{1}{\sqrt{3}}$

$v = \frac{\pi}{6}$ $\quad y = -\frac{\pi}{6}$

$r = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$* \quad f(x) = 2\cos\left(-\frac{\pi}{6}\right)\sin x + 2\sin\left(-\frac{\pi}{6}\right)\cos x = 2\sin\left(x - \frac{\pi}{6}\right)$

STÖRSTA VÄRDET $f(x)$ ANTAR ÄR 2 OCH
MINSTA " " " " -2 TY $\left|\sin\left(x-\frac{\pi}{6}\right)\right| \leq 1$.

STÖRSTA VÄRDET FÖR SIN X ANTAS DÅ $x = \frac{\pi}{2} + n \cdot 2\pi$
MINSTA " " " " " $x = -\frac{\pi}{2} + n \cdot 2\pi$

NÄR ÄR $x - \frac{\pi}{6} = \frac{\pi}{2}$? $\iff x = \frac{\pi}{6} + \frac{\pi}{2} = \frac{4\pi}{6} = \frac{2\pi}{3}$

OCH FÖR VILKA X ÄR $x - \frac{\pi}{6} = -\frac{\pi}{2} \iff x = \frac{\pi}{6} - \frac{\pi}{2} = -\frac{2\pi}{6} = -\frac{\pi}{3}$

SVAR! STÖRSTA VÄRDET 2 ANTAS DÅ $x = \frac{2\pi}{3} + n \cdot 2\pi$

MINSTA VÄRDET -2 ANTAS DÅ $x = -\frac{\pi}{3} + n \cdot 2\pi$

9. VISA $\sum_{k=1}^{n} (2k+1)3^k = n \cdot 3^{n+1}$

$P(n)$: $3 \cdot 3^1 + 5 \cdot 3^2 + 7 \cdot 3^3 + \cdots (2n+1)3^n = n \cdot 3^{n+1}$

$P(1)$! $VL = 3 \cdot 3^1 = 9$ $HL = 1 \cdot 3^2 = 9$ $VL = HL$ ∴ $P(1)$ SANN

ANTAG $P(n)$ SANN

VISA $P(n+1)$:

$3 \cdot 3 + 5 \cdot 3^2 + \cdots + (2n+1) \cdot 3^n + (2(n+1)+1)3^{n+1} =$

$= \left|\text{ENL. INDUK. ANTAGANDET}\right| = n \cdot 3^{n+1} + (2n+2+1)3^{n+1}$

$= (n + 2n + 3) \cdot 3^{n+1} = (3n+3) \cdot 3^{n+1} = 3(n+1) \cdot 3^{n+1}$

$= (n+1) 3^1 \cdot 3^{n+1} = (n+1) \cdot 3^{n+2}$

∴ $P(n+1)$ SANN.

$P(n) \Rightarrow P(n+1)$ SANN

$\Rightarrow$ $P(n)$ SANN FÖR ALLA $n \geq 1$ ENLIGT INDUKTIONSPRINCIPEN.